1. Introduction

The vast amount of data and the increasing development in technology in recent years have changed the way in which many industries operate and compete with each other. Millions of bytes, commonly referred to as big data, provide valuable insights for companies to make informed business decisions. Companies that conduct business in the financial service sector employ big data to inform their investment practices and make strategic decisions. The increased use and complexity of big data poses a challenge to users of financial information when analyzing financial statements. This is especially applicable to users who possess fewer financial resources and have inferior knowledge to conduct in-depth analysis of financial statements (Lokanan, 2014). Companies that want to present a rosy picture of their financial position, may exploit these users’ deficiencies through deliberate misleading and omission of financial data in their annual reports (Rezaee, 2002; Albrecht et al., 2006; 2014; Robinson and Lokanan, 2017).

Vietnamese companies were selected because of the high incidence of financial reports manipulation (Tran, 2013). The number of listed companies reported by Hanoi Stock Exchange (HNX) and Ho Chi Minh City Stock Exchange (HOSE) from 2000, when Vietnam’s security market was in its infancy stage, to 2016 has steadily increased. In 2016, there were more than 1,000 listed companies on these exchanges. Growth and structural development in Vietnam’s financial markets comes with intense competition in the marketplace and the possibility of financial statement manipulation of listed companies on the HNX and HOSE (Tran, 2013). Indeed, there has been an increasing number of failed companies and fraudulent reporting in Vietnamese markets in the last few years. To be specific, 6,608 companies in the first seven months of 2017, 12,478 companies in 2016 and 9,467 companies in 2015 (Agency of Business Registration, 2017). The volume and intensity of fraudulent reporting have made it difficult for humans to process and analyze anomalous transactions (Grace et al., 2017). Even some traditional statistic regression techniques cannot be applied due to the complexity of data set (Fan and Li, 2006). Thus, we need embedded analytical models with highly-automated operating structures to deal with the large volume, variety of features, and velocity of the data that the human brain cannot handle.

This is where big data techniques come into play. Big data have brought with it novel techniques, such as machine learning and algorithms, that allow users to conduct in-depth analysis and gain deeper understanding of anomalies in financial statements. The analysis of big data using machine learning techniques can assist users of financial statements to detect unusual patterns and transactions in companies’ financials. Big data are massive and can be used by both users and companies to provide data-centric and data-driven insights on financial statement anomalies.

This study is an attempt to use machine learning algorithms to detect anomalies in financial statements in Vietnamese listed firms. As mentioned, the only resources available to ordinary investors are quarterly reports, which may contain misleading financial information. It is not enough just to look at the original state of such financial reports. Much research has proved efficiency by analyzing financial ratios calculated from the values in companies’ reports (see Altman, 1968; Kotsiantis et al., 2006; Pustylnick, 2011). Therefore, we
approached the problem by using financial ratios as a series of variables, also known as features. An important point in this paper is that the values of financial ratios are assumed to follow a multivariate distribution, which means each ratio varies around one specific mean value. This assumption will allow us to point out anomalous data by measuring whether the distance of each datum to the ‘centroid’ (which will be explained in Research Methodology) exceeds a certain threshold. Additionally, we will take the concept of distance further by regarding it as the degree or extent of the anomaly. This extension of understanding enables us to rank the credit worthiness of each company in each quarter: the more anomalous a datum, the less credit-worthy it is. Therefore, the central question of this paper is as follows: is it possible to rate the creditworthiness of a firm’s financial quarter using an anomaly detection method?

It is also worth noting that, up to this point, we have used the term ‘anomaly’ instead of ‘fraud’ for the main theme of this paper. There is a slight difference between the two terms: ‘anomaly’ generally represents “an unusual and possibly erroneous observation that does not follow the general pattern of a drawn population” (Morozov, 2016, p. 63), while ‘fraud’ is an intentional deceptive action perpetuated against a firm for financial gains (Lokanan, 2015). Because we did not have enough data about fraudulent companies or illegal activities in Vietnam, we chose to use ‘anomaly’ for simplicity and precision. However, we still looked at the fraud detection literature, as it points to a more expansive understanding of existing analytical techniques.

The rest of the paper proceeds according to the following format. We first present a comprehensive review of financial fraud detection research using machine learning methods. In this regard, we provide an analysis of the existing fraud detection literature based on the most important machine learning algorithms and statistical methods employed in the literature to date. Next, we outline the methodology and research design used to collect and analyze the data. This is followed by an analysis of the empirical findings. Finally, we present a conclusion and highlight some of the key issues associated with current practices and highlight areas for future research.

2. Contribution to Theory and Practice

Theoretically, the paper provides guidance on the machine learning technique and algorithms to use when creating new models for detecting anomalies in financial statements. Statistical methods have been the go-to method to evaluate information from financial statement reports (Agarwal and Taffler, 2008; Altman et al., 2013; Tinoco and Wilson, 2013; Lokanan, 2017). While a statistical method has been a success in detecting anomalies in prior research (Beneish, 1997; Bell and Carcello, 2000; Lyandres and Zhdanov, 2013), machine learning techniques have proven to be just as or even more effective in classification performance (Feroz et al., 2000; Lin et al., 2003; Kotsiantis et al., 2006; Perols, 2011) because they make for easier processing of large data (Fan and Li, 2006) and have proven to be neutral in decision-making (West et al., 2005; Kotsiantis et al., 2006). Traditional statistical analysis produces errors terms when there are many observations and the use of longer time series data into the models (Feroz et al., 2000; Lin et al., 2003; Lokanan and Sharma, 2018).

Practically, machine learning, with its many features, can allow users to handle large datasets and improve upon statistical models (Kotsiantis et al., 2006; Perols, 2011). When approaching a problem through traditional statistical methods, the human mind can come up with many hypotheses and generate false prediction towards existing information, because
hypotheses are never entirely accurate (Farber, 2005; Purda and Skillicorn, 2015). The use of
a machine learning tool will reduce the number of hypothesis tests in the calculation by using
only primary input data. In this regard, machine learning algometric technique will address
the shortcomings of hypothesis bias in traditional statistical data analytics (Perols, 2011).

The results presented in this paper are one step closer to heed the calls for the
acceleration of technology in analyzing financial statement data and lays the groundwork for
further research on the automation of fraud detection (Kirkos et al., 2007; Song et al., 2014;
Lokanan, 2015). It is expected that the models employed in this paper will aid investors and
other users of financial information to use financial data, even the most technical ones, to
conduct informed analysis of companies’ financial performance. The models are built from
algorithms and a very large data set, which, together, can inform users’ intelligence about red
flags of fraud in financial statements.

3. Prior Research

3.1. Financial Statement Irregularities

Over the past three decades, there has been an increased focus on irregularities in
corporate accounting reporting in general and financial statement fraud in particular (Beasley,
1996; Beneish, 1997; 1999; Rezaee, 2005; Hogan et al., 2008; Cooper et al., 2013; Lokanan,
2015; Morales et al., 2014). Generally, the literature on financial statement fraud focuses on
the individual factors that affect fraudulent behavior in organizations (Albrecht et al., 2004;
Bell and Carcello, 2000; Rezaee, 2005; Dellarotass, 2013); the procedures and expertise of
auditors to detect "red flags" of fraud (Albrecht and Albrecht, 2004; Rezaee, 2005; Murphy
and Dacin, 2011; Murphy, 2012; Power, 2013; Morales et al., 2014); the effects of fraud risks
assessment tools on high risks areas in audit engagements (Johnstone and Bedard, 2001;
Rezaee, 2005; Davis and Pesch, 2013; Power, 2013; Lokanan, 2015; Behzadian and Izadi Nia,
2017); and, the role of auditing committees to detect red flags associated with fraud
(Johnstone and Bedard, 2001; Kranacher et al., 2010; Lokanan, 2014). Together, the academic
research offers insights on financial statement fraud and facilitates the development and
enhancement of new technologies to detect anomalies in fraud (Hogan et al., 2008; Albrecht
et al., 2014; Morales et al., 2014).

3.2. Detecting Anomalies in Financial Statements Using Machine Learning

Financial statement fraud is an issue with far reaching consequences (Rezaee, 2005;
Albrecht et al., 2014; Lokanan, 2015). Traditional methods involving manual detection, while
successful in certain areas (Eining et al., 1997; Farber, 2005; Beneish et al., 2013; Hajek and
Henriques, 2017; Lokanan, 2017), are not only time-consuming and expensive, but, in the
age of big data, they are also impractical and unable to deal with large volumes of
unstructured data (Perols, 2011). Largely driven by an increase in instrumentation, the
financial service industry has turned to automated processes using statistical and
computational methods to analyze financial statements data (Anandakrishnan et al., 2017).
Machine learning algorithms are not only useful in dealing with big data, they can also mimic
how users process unstructured data, text, speech, and image, to improve accuracy in
interpreting financial statements (Feroz et al., 2000; Beneish and Craig, 2007; Song et al.,
2014).

Research that evaluates the effectiveness between machine learning and traditional
statistical methods typically compares fraud classification algorithms with regression models (Green and Choi, 1997; Lin et al., 2003; Kirkos et al., 2007; Perols, 2011; Morozov, 2016; Lokanan and Sharma, 2018). This stream of research employed logistic regression, artificial neural networks (ANN), fuzzy logic, and ensemble-based methods and found that the techniques combined are useful for fraud detection even when fraud cases are rare or unavailable. The distinguishing elements that make this stream of research unique, however, is that there are many more companies that prepare accurate financial statements than those that falsify their financial statements (Perols, 2011). The attributes (i.e., financial ratios) used to classify the fraud are noisy, thereby making companies that falsify their financial statement look similar to companies with accurate and clean financial statements (Lin et al., 2003; Kotsiantis et al., 2006; Kirkos et al., 2007; Purda and Skillicorn, 2015).

Research using logistic regression models has found it to be rather unique when comparing fraud and non-fraud firms (Lin et al., 2003; Kirkos et al., 2007; Hajek and Henriques, 2017; Lokanan and Sharma, 2018). Bell and Carcello (2000) employed a logistic regression model that estimates the likelihood of fraudulent financial reporting. Using a sample of 77 fraud engagements and 305 non-fraud engagements, Bell and Carcello (2000) found that their logistic regression model was significantly more accurate than practicing auditors in assessing the risks of fraud for the 77 observations. In another study, Lokanan and Sharma (2018) employed a logistic regression model to test for red flags of fraud in banks that were involved in the LIBOR scandal. Using financial ratios and corporate governance data relating to the sixteen banks that were involved in the LIBOR scandal with a matched sample of non-fraud banks, the authors found supports for using financial ratios and governance data to detect fraud in banks (Lokanan and Sharma, 2018). In a similar study, Skousen et al. (2015) employed a logistic regression model to detect financial statement fraud between a set of fraud firms and a matched sample of non-fraud firms. The study revealed that the logistic regression model was effective in predicting which of the sample firms committed fraud versus those that did not. Likewise, Spathis (2002) found that multivariate logistic regression techniques were accurate in detecting false financial statements, using a sample of fraud and non-fraud firms. In another study, Lin et al. (2003) found that fuzzy neural network (FNN) outperformed logistic regression model and ANN in the prediction of fraud cases. Hajek and Henriques (2017) also found that logistic regression was also useful in detection of financial statement fraud.

Another stream of research focusing on evaluating the classification of machine learning algorithms in detecting fraud in financial statement typically used different variations of ANN (e.g., Eining et al., 1997; Green and Choi, 1997; Fanning and Cogger, 1998; Lin et al., 2003). Green and Choi (1997) showed that there is support for ANN as a fraud-risk assessment tool. Other studies found a high probability of detecting fraudulent financial statements using ANN rather than probit or logit models (Eining et al., 1997; Fanning and Cogger, 1998). Feroz et al. (2000) illustrated the application of ANN to test the ability of selected Statement of Auditing Standards (SAS) No. 53 to predict the targets of the Securities and Exchange Commission’s (SEC) investigations and found that an analysis of financial ratios from the trial balance does have predicted value. Harymawan and Nurillah (2017) employed a multiple regression model to test for earnings management in financial reporting and found that corporate reputation has a significant relationship with earnings quality. These studies reinforced the efficiency for using machine learning algorithms as suggested techniques to detect anomalies in financial statements.

Other research looked at classification algorithms to improve fraud classification
performance (Kotsiantis et al., 2006; Kirkos et al., 2007; Perols, 2011). These studies explored the effectiveness of machine learning algorithms to detect firms that issue fraudulent financial statements through various algometric classifications. Kotsiantis et al. (2006) employed a sample of fraud and non-fraud firms and financial ratios to examine the following classification algorithms: S, K2, C4.5, 3NN, RBF, Ripper, LR, and SMO. The findings revealed that the algorithmic classifications performed better than logistic regression and ANN models. Kirkos et al. (2007) classified algorithms into Decision Trees, ANN, and Bayesian Belief and examined their usefulness to detect fraud in financial statements. Using financial statement ratios, the study employed a sample of fraud firms with a matched sample of non-fraud companies and found that the Bayesian Belief outperforms Decision Trees and ANN in financial statement fraud detection. In another study, Hoogs et al. (2007) presented a genetic algorithm approach to detecting financial statement fraud. Using a sample of fraud companies accused of improper revenue recognition by the SEC and a matched sample of non-fraud companies, the study found that genetic classification of algorithms has many features well-suited for accurate fraud detection. More recently, Dbouk and Zaarour (2017) employed a Bayesian Naïve Classifier (BNC), a supervised machine learning approach, and found that the BNC’s approach outperforms conventional audit method in detecting earnings manipulations.

Machine learning research has developed ensembles of predictors other than ANN and logistic regression to generate hypothesis for testing in fraud research (Perols, 2011; Phua et al., 2004). The research in this category used cluster algorithm (i.e., K-means clustering) (Li, 2016); bagging (West et al., 2005), and support vector machine (SVM) (Shin et al., 2005) to assess anomalies in financial fraud. Of particular interest with this stream of research is that they used ensemble methodology to show that anomaly detection and predictive analytics for financial risk management bring out the ideas of using some algorithms together. In general, ensemble predictors were found to be superior to single machine learning and statistical models for detecting fraud in financial statements (Phua et al., 2004; West et al., 2005). Ensemble predictors are also able to extract optimal solutions with small and noisy dataset (Fan and Palaniswami, 2000; Shin et al., 2005).

The foregoing literature review highlighted the various statistical techniques and machine learning models used to identify anomalies in financial statements. With respect to fraud detection, there is a significant body of research that provides support for machine learning and, to a certain extent, logistic regression models (Chen and Rezaee, 2013; Albrecht et al., 2014). Overall, ANN outperforms logistic regression models in the literature; however, both were found to be inferior when compared to ensemble-based and classification algorithms methods. Despite this authoritative guidance on these statistical models and machine learning techniques, there remains a significant gap between fraud detection models and their application to large volume of time series and cross-sectional data. This study is an attempt to address these gaps by using machine learning techniques to detect red flags of fraud on a sample of Vietnamese companies.

4. Research Methodology

4.1. Data Source and Collection

In this study, we use financial statement ratios to build the algorithms. The financial ratios, which are divided into seven groups, are obtained from Cophieu68 and Vietstock. These sources contain quarterly and annual financial reports of all listed companies on the
Vietnamese stock market from 2011 to 2016. We used data for this period because it was the period when the stock exchange in Vietnam had the largest volume of readily available data (i.e., not too many missing data). The data used in this study contain both audited and un-audited quarterly reports. Quarterly reports are not legally required to be audited in Vietnam. Unaudited raw data have the advantage of showing the earliest anomalous situation in financial statements.

Data were collected from the most reliable sources available in Vietnam: income and cash flow statements from Cophieu68.vn and balance sheets from Vietstock finance. After having excluded banks, financial, insurance companies as well as recently merged or acquired firms, we obtained a total of 937 listed Vietnamese firms. Each document for a company was stored in a matrix-like data structure whose columns are the indices and rows are observations. We chose to conduct anomaly detection on a quarterly basis as we wanted the result to eliminate the large possible time lag. Also, audited information could be booked, thus covering the real financial situation of the companies. The timeframe of the data is from Quarter 1 - 2011 to Quarter 4 - 2016, which spanned 24 quarters. The data values in each quarter represent an observation. We were able to extract data from 1,090 companies listed on Vietnam’s stock exchanges. However, 153 financial institutions were eliminated from the sample due to their unique form of financial statements and business operation. The final data set consisted of 937 companies and 22,488 observations.

4.2. Data Pre-processing
4.2.1. Financial indices calculation

We identified that there were 31 essential financial indices. However, we excluded seven indices because of high correlation, whose thresholds were greater than 0.8, to avoid multi-collinearity issues. Thus, we obtained 24 indices that can be considered independent. Every index in the seven categories listed below has different implications for detecting financial anomalies:

1. Liquidity ratios: used to determine how quickly a company can turn its assets into cash if it is experiencing financial distress or impending bankruptcy.
2. Profitability ratios: are ratios that demonstrate how profitable a company is.
3. Activity ratios: are meant to show how well management is managing the company's resources.
4. Solvency ratios: depict how much a company relies upon its debt to fund operations.
6. Accrued income: is earned in a fund or by a company for providing a service or selling a product that has yet to be received.
7. Cash flow: is the net amount of cash and cash-equivalents moving into and out of the business.

The general overview of data collection process to obtain financial indices is illustrated in Figure 1.
4.2.2. Data normalization

Due to different sizes of the companies, their financial indices are on various scales. Without normalizing, the model will be biased. In order to proceed, it is necessary to mention vector operations on the matrix-like data structure. In Figure 1 above, each row is considered a vector. A vector can contain one or many components, and, in this particular case, there are 24 components, which are the financial indices obtained from the data collection process. Thus, when we say we conduct a vector operation on two vectors, such as summing two vectors, we are just summing each corresponding component of two vectors to create a new vector with the same number of components. Vector operations are powerful and essential to perform gigantic calculations on matrix-like data structure.

In the scope of our research, we chose standardization as the method of normalizing our training and testing data because we assumed that, over time, financial indices of a company would stabilize and follow the standard normal distribution, which later will be denoted as “$\sim N(0,1)$”. In the standardization method, we compute the values of each financial index to have zero mean and unit variance. First, we calculated the mean and standard deviation of the range of feature values. Next, we subtracted the mean from each function’s value, then divided the result by the standard deviation. The process is summarized in the following formula:

$$x' = \frac{x - \mu}{\sigma} \quad \text{(Equation 4.1)}$$

To implement this, we conduct the following steps:
First: Obtaining the mean vector (MU) which represents the mean value of all financial indices.

Second: Obtaining the standard deviation vectors (Std) of each company from the training data set, which will be described in detail in the next section.

Third: We conduct a matrix operation according to equation (4.1), with \( x \) as each observation’s vector, \( \mu \) as the mean vector and as the standard deviation vector of each company. By doing this, we can obtain normalized data values for every observation as \( x' \).

**Table 1:** A visualization of the mean vectors of the companies

<table>
<thead>
<tr>
<th>Companies</th>
<th>Financial indices</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Index 1</td>
<td>Index 2</td>
</tr>
<tr>
<td>Company A (vector A)</td>
<td>Mean of index 1(A)</td>
<td>Mean of index 2(A)</td>
<td>…</td>
</tr>
<tr>
<td>Company B (vector B)</td>
<td>Mean of index 1(B)</td>
<td>Mean of index 2(B)</td>
<td>…</td>
</tr>
</tbody>
</table>

### 4.2.3. Pre-implementation

Before implementing the models, we add some fine-tunings for missing data. For a company when there is completely no available data for a financial index, that financial index is removed from the computation. Also, if values of a financial index are partially missing (denoted in the data set as ‘N/A’), we replace them with values generated from the standard normal distribution. Since we investigate anomalies on a company basis, the procedure for a company does not affect the data values of other companies.

It can be argued that, for every time a missing value is filled with a value from the standard normal distribution, the result will be different. However, we must also be careful not to fill in empty values with a fixed value, such as 0 (a common solution), because the resulting data values will not align with the assumed distribution. A possible solution for this issue is that, for every random value that needs to be filled in, we can set a ‘random state’, which will cause the randomized function to always return the same random number for every run. In doing so, interested readers or researchers can simulate the same implementation to understand the findings and replicate the models in future research. Table 2 presents a preview of the normalized training data frame.
5. **Research Design**

5.1. **Multivariate Normal Distribution and Assumptions**

Since the data have 24 independent financial indices, which correspond to 24 features, the multivariate normal distribution (MVN) will be implemented in our model. In general, it is the generalization of the univariate normal distribution to multiple variables (Fan and Palaniswami, 2000; Lokanan, 2017). Although real data may never come from a right MVN, the MVN provides a robust approximation and has many desirable mathematical properties, such as the mean vector and covariance matrix. Furthermore, because of the central limit theorem, many multivariate statistics converge to the MVN distribution as the sample size increases. Overall, MVN has the following properties:

- Joint density.
- Shape: The contours of the joint distribution are \( n \)-dimensional ellipsoids.
- Mean, and covariance specifies the distribution. The \( \text{MN}(\mu, \Sigma) \) joint distribution is determined by \( \mu \) and \( \Sigma \) only.
- Moment generating function: The \( \text{MN}(\mu, \Sigma) \) distribution
  \[
  MGF_M(t) = \exp \left( \mu^T t + \frac{1}{2} t^T \Sigma t \right) \quad \text{(Equation 5.1)}
  \]
  where \( t \) is a real \( n \times 1 \) vector.
- Characteristic function: The \( \text{MN}(\mu, \Sigma) \) distribution has
  \[
  CF_{\psi}(t) = \exp \left( \mu^T t - \frac{1}{2} t^T \Sigma t \right) \quad \text{(Equation 5.2)}
  \]
  where \( t \) is a real \( n \times 1 \) vector.
- Linear combinations.
- Independence.

With MVN, the following assumptions are made regarding our research case:

- Indices after calculation and scaling are Gaussian independently distributed. If not, they are either transformed or omitted from the model.
- Some financial indices are too important to be overlooked. As such, we retained them even if they were highly correlated with the others.
- Ninety-five percent of “none - anomalous” data points stay within \([ \mu - 3\sigma; \mu + 3\sigma] \)
- Company indices change slowly and can be inferred from history and other indices.

5.2. **Mahalanobis Distance - Statistical Distance**

One of the simplest methods to filter anomalous observations is the Mahalanobis distance ((Thongkam et al., 2008). The Mahalanobis distance will be employed in this study to decide whether an observation is anomalous. Conceptually, the Mahalanobis distance measures the proximity of a data point to the center of the distribution and is a direct generalization of standard deviation. The Mahalanobis distance of a point \( x = (x_1, x_2, \ldots, x_n) \) is defined as follows:
In equation (5.3), \( \mu = (\mu_1, \mu_2, \ldots, \mu_n) \) is the mean vector of the distribution; \( \Sigma \) is the covariance matrix of the features in the n-dimensional space. For the application of anomaly detection, we will measure the distance of each test observation to the mean vector \( (\mu) \) of each company’s data in standard deviation unit. The formula is computed as follows:

\[
d_M(x, \mu) = \sqrt{(x - \mu)^T \Sigma (x - \mu)^{-1}} \quad \text{(Equation 5.4)}
\]

It is assumed that each feature (financial index) \( \sim N(0,1) \), each observation vector, \( x \sim N_p(\mu, \Sigma) \) with \( \Sigma \succ 0 \) and \( p \) is the number of features. Therefore, the random variable \( D = (x - \mu)^T \Sigma (x - \mu)^{-1} \) has the Chi-squared distribution with \( p \) degrees of freedom (Bajorski, 2011). From this property, it is inferred that:

\[
P(d_M(x, \mu) \leq k) = G_p(k^2) \quad \text{(Equation 5.5)}
\]

where \( G_p \) is the Cumulative Distribution Function (CDF) of the Chi-squared distribution with \( p \) degrees of freedom.

By choosing the value of \( k \) and referring to the Chi-squared distribution table, we can obtain the probability of an observation having its distance to the mean vector less than \( K \)-standard deviations. This probability is also the proportion of data observations having their distances to the mean vector less than \( k \) standard deviations. The application for anomaly detection, which will be discussed in the next part, is mostly based on the conclusion above.

5.3. Anomalies Detection with Mahalanobis Distance

After collecting data, a correlation matrix was formulated for each company. This approach allows us to measure the correlation between the companies’ indices. We also categorized the original data set into two parts: 83% for training and 17% for testing. Table 3 shows that the one-dimension (1-D) tensors or vectors are represented for calculating Mahalanobis distance:

**Table 3**: Input: 1-D tensors ~ vectors as below for calculating:

<table>
<thead>
<tr>
<th>Company A’s current ratio</th>
<th>Company A’s quick ratio</th>
<th>..</th>
<th>Company A’s total net accruals</th>
</tr>
</thead>
</table>

\[
f(x_1, x_2, \ldots, x_k) = \frac{1}{\sqrt{(2\pi)^k |\Sigma|}} exp \quad \text{(Equation 5.6)}
\]

This function can be generalized for the vector of 1 row \( \times 24 \) columns (1×24). However, to visualize the Multivariate Gaussian Distribution, we will use a vector 1×2, as shown in Table 4.

**Table 3**: 1×2 matrix for better visualization on three-dimension space

<table>
<thead>
<tr>
<th>Company A’s current ratio</th>
<th>Company A’s quick ratio</th>
</tr>
</thead>
</table>
[Current Ratio, Quick Ratio]

The visualization of a 1-D tensor with the size of 1×2 example can be seen in Figure 3. Figure 2. The visualization of a 1-D tensor with size 1×2

The contour of this bivariate normal distribution is visualized in Figure 3. Importantly, when we squash three-dimension data points into two-dimension ones, the dataset will lose valuable information. As we can see in the function and visualization in Figure 3, the output of the MVN is a learned model from the training data set, while \( f(X) \) is the probability of whether a data point is part of the normal distribution. We have to set a value of epsilon (\( \epsilon \)) to compare with \( f(X) \). If \( f(X) < \epsilon \), the data point is anomalous, and vice versa.
However, because of computational complexity, we will not deploy calculations on \( f(X) \) directly. Instead, we deal with MVN in another way: given the assumption of the central limit theorem, we will calculate the loss, the Mahalanobis distance, as the product between test data point and the mean. If the loss \( \geq 3\sigma \), we can conclude the level of anomalies is high, and vice versa. As mentioned above, we can calculate the Mahalanobis distance for each observation. If any of these distances is greater than a certain threshold \( L_{max} \), we consider that observation an anomaly. To calculate \( L_{max} \), we check how likely it is that the most significant Mahalanobis distance is greater than \( L_{max} \) using the following equation:

\[
Y_i = (X_i - \mu)^T - \Sigma(X_i - \mu)^{-1}, i = 1, \ldots, n \quad \text{(Equation 5.7)}
\]

From the section Mahalanobis Distance - Statistical Distance, each \( Y_i \) follows the Chi-squared distribution with \( \rho \) features - \( \rho \) degrees of freedom. Now, we can calculate the probability that the largest \( Y \) is larger than \( L^2 \) using the following equation:

\[
P(\max_{1 \leq i \leq n} Y_i > L^2) = 1 - P(\max_{1 \leq i \leq n} Y_i \leq L^2) = 1 - \prod_{i=1}^{n} P(Y_i \leq L^2) = 1 - \left[ G_\rho(L^2) \right]^n \quad \text{(Equation 5.8)}
\]

where \( G_\rho \) is the CDF of the Chi-squared distribution with \( \rho \) degrees of freedom.

For this probability to be equal to a small value, we need:

\[
L = \sqrt{\chi^2_\rho(1 - \alpha)^{\frac{1}{n}}} \quad \text{(Equation 5.9)}
\]

In this study, we choose the value to be equal to 5\%, which is a traditionally preferred value when refining significance level in academics (Torbeck, 2010). Any observation whose distance to the mean vector is greater than this value of \( L(\alpha = 0.05) \) will be considered an anomaly. As mentioned earlier, we set the fixed \( L_{max} \) with \( p = 95\% \) and 24 features as 2.64E+15. If the responding Mahalanobis distance of a specific quarter is greater than the \( L_{max} \) value, we can be sure that the quarters of the specific company's financial indices were anomalous. Furthermore, as mentioned in the Introduction, we consolidate the distances into ordered categories to rank the companies’ credit-worthiness. The projected anomaly ratings are defined in Table 5 below.

Table 4. Company rating types for all observed companies
6. Empirical Results

Training classification was formed to display the data (Perols, 2011). Out of the 22,488 observations, the training group consists of 18,740 observations, while the test group consists of 3,748 observations (e.g., see Spathis, 2002; Lin et al., 2003; Kirkos et al., 2007; Lokanan and Sharma, 2018). We first employ the Mahalanobis distance and consider each firm in a quarter as a data point. The summary of the Mahalanobis distance of the testing data set is shown in Table 6 below. The mean and the standard deviations for the distance from one datum to central limit come from the 3,748 observations. The mean distance measuring average length from firm-quarter data point to the center was 5.65E+20. The maximum and minimum Mahalanobis distances from the testing data set are 3.1E+21 and 1.25E+24, which represent observations with the largest and smallest degree of anomaly, respectively. These findings indicate that, for the most part, a significant proportion of the companies were within the normal (versus ‘anomaly’) standard deviation range in their financial statements. A closer look at the results in Table 6 shows that most of the observations were closer to the mean and that there was not much variance (i.e., larger standard deviations) among companies.

Table 5. A summary of Mahalanobis Distance

<table>
<thead>
<tr>
<th>Type</th>
<th>Definition</th>
<th>$d_M$ range</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Good company with no or few anomalies, predictable outcome</td>
<td>[0; 1E+15]</td>
</tr>
<tr>
<td>B</td>
<td>Normal company with average anomaly, quite predictable outcome</td>
<td>(1E+15; 5E+15]</td>
</tr>
<tr>
<td>U</td>
<td>Unranked company with many anomalies, unpredictable outcome</td>
<td>(5E+15; +infinity)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of observations</th>
<th>Mean</th>
<th>Std</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>3748</td>
<td>5.65E+20</td>
<td>2.49E+22</td>
<td>-3.1E+21</td>
</tr>
<tr>
<td>Top 25%</td>
<td>Top 50%</td>
<td>Top 75%</td>
<td>Max</td>
</tr>
<tr>
<td>-2.4E+15</td>
<td>0.612693</td>
<td>3.69E+15</td>
<td>1.25E+24</td>
</tr>
</tbody>
</table>

For more details, the company ratings defined above can be applied to every observation or firm-quarter in the dataset. As an example, Tables 7, 8 and 9 below show the Mahalanobis distance values by quarters for Vietnamese listed companies represented by their respective tickers for 2016. As can be seen in Table 7, for the entire 2016 financial year, Type A companies from stock ticker AAA had a small number of anomalies in their financial statements. These results were predicted as it was expected that companies with A ratings will

<table>
<thead>
<tr>
<th>Type</th>
<th>Definition</th>
<th>$d_M$ range</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Good company with no or few anomalies, predictable outcome</td>
<td>[0; 1E+15]</td>
</tr>
<tr>
<td>B</td>
<td>Normal company with average anomaly, quite predictable outcome</td>
<td>(1E+15; 5E+15]</td>
</tr>
<tr>
<td>U</td>
<td>Unranked company with many anomalies, unpredictable outcome</td>
<td>(5E+15; +infinity)</td>
</tr>
</tbody>
</table>
have small variance in their financial statements. On the other hand, with the exception of the first quarter, the unranked companies in stock tickers AAM (Table 8) had a higher number of anomalies with very unpredictable outcomes. The companies in the stock ticker DGH (Table 9) had fluctuating results throughout the 2016 fiscal year. For quarters two and four, there was very little anomaly detection and a strong indication that companies were producing reliable financial statements. In quarter three, the results from the Mahalanobis distance reveal the companies had average anomalies and quite predictable outcome. For the unranked companies in the DHG stock ticker, there were many anomalies with very unpredictable outcome in the first quarter of 2016. Overall, it may not be to a company’s advantage to manipulate their financial statements, especially when the company already has a good reputation (Harymawan and Nurillah, 2017)

Table 6. Financial anomaly analysis result of stock ticker AAA

<table>
<thead>
<tr>
<th>Time</th>
<th>Mahalanobis Distance</th>
<th>Anomaly Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>First quarter 2016</td>
<td>-3.1E+15</td>
<td>A</td>
</tr>
<tr>
<td>Second quarter 2016</td>
<td>3.7E+14</td>
<td>A</td>
</tr>
<tr>
<td>Third quarter 2016</td>
<td>2.7E+14</td>
<td>A</td>
</tr>
<tr>
<td>Fourth quarter 2016</td>
<td>-1.4E+16</td>
<td>A</td>
</tr>
</tbody>
</table>

Table 7. Financial anomaly analysis result of stock AAM

<table>
<thead>
<tr>
<th>Time</th>
<th>Mahalanobis Distance</th>
<th>Anomaly Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>First quarter 2016</td>
<td>3.19E+14</td>
<td>A</td>
</tr>
<tr>
<td>Second quarter 2016</td>
<td>2.15E+16</td>
<td>U</td>
</tr>
<tr>
<td>Third quarter 2016</td>
<td>1.48E+16</td>
<td>U</td>
</tr>
<tr>
<td>Fourth quarter 2016</td>
<td>3.44E+16</td>
<td>U</td>
</tr>
</tbody>
</table>

Table 8. Financial anomaly analysis result of stock ticker DHG
Table 10 shows the number of rated firm-quarter. A closer look at Table 6 shows that 68.89% of firm-quarter data is rated A. This means that most of the companies were performing very well with few anomalies in their financial statements. Considering the concerns regarding fraudulent financial statements in Vietnam, these findings are significant for two reasons. First, it shows that there are no overall financial statement level threats and users can feel confident in using these financial statements to make informed financial decision. In other words, the findings reduced overall financial statement level (OFSL) risk, that is, the risk of material misstatement for the financial statement as a whole (see also Behzadian and Izadi Nia, 2017). Second, and partly synonymous with the first, is that the results directly enhance the relevance and reliability of information provided in the financial statements for strategic investment decision-making. The rated B companies account for about 7.6% of anomalies. This is a minimal amount compared to 23.51% of unranked firm-quarters that have significant anomalies with very unpredictable results. This is a serious concern and Vietnamese regulators need to take stock of these results. That fact that 23.51% of unranked companies have significant anomalies in their financial statements raises serious questions concerning the efficiency of the audit approach and procedures used to audit these financial statements.

Table 9. The summary of company ratings

<table>
<thead>
<tr>
<th>Type</th>
<th>Number of companies</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of type A - rated firm-quarter</td>
<td>2582</td>
<td>68.89%</td>
</tr>
<tr>
<td>Number of type B - rated firm-quarter</td>
<td>285</td>
<td>7.6%</td>
</tr>
<tr>
<td>Number of type U - rated firm-quarter</td>
<td>881</td>
<td>23.51%</td>
</tr>
</tbody>
</table>

Figure 4. Visual breakdown of companies in each rating type (in percentages)
7. Conclusion, Limitations and Areas for Future Research

The paper produces two meaningful outcomes regarding anomalies detection and anomaly rating of financial statements. First, a significantly high proportion (68.89%) of Type A firms had very few anomalies and this is a healthy sign that they are in compliance with legal and ethical standards. For a user’s perspective, they can feel confident in using these financial statements to make investment decisions. Second, and more concerning, is that 23.51% of the unranked companies had significant anomalies in their financial statements. This is a worrying sign as it indicates that there are risks in the financial statements as a whole and there is a likelihood of fraud or error in the financial statements. In both cases, the first outcome laid a solid foundation for users to analyze and understand financial statements, while the second outcome indicates that, as an aggregate, regulators need to start paying more attention to the audited financial statements of unranked companies for potential fraud or error.

From a practical perspective, the results presented here can be used to provide guidance to audit committees and senior executives concerned about the increased possibility to deter potential accounting fraud or misstatements in financial reporting. Just as there is a need for higher ethics in organizational research, so too is the need for managers to recognize the importance of the fraudulent mechanics of an imperfect system that promotes unethical financial reporting (Lokanan, 2015). If, as seems to be the case, unethical behavior by managers gives a competitive advantage to a company, then monitoring their activities becomes an important strategic objective for firms and external auditors to report them to regulators and publicize the irregularities to all users of their financial information.

People working in the corporate sector regularly spend a lot of time in financial districts, which, despite the best regulatory efforts by regulators, offer regular opportunities to engage in fraud and/or experience sources of friction, which may lead to fraud (Lokanan, 2015; Morales et al., 2015). An accountant, for example, may address the friction caused from not meeting financial targets by manipulating the financial statements (Behzadian and Izadi Nia, 2017; Lokanan, 2018). A chief executive officer (CEO) may address the friction caused from not meeting financial targets by manipulating his compensation bonuses (Perols, 2011). As these individuals repeat their behaviors, questions arise over the materiality of the OFSL risks in these statements. The model employed in this paper is capable of addressing these issues in the specific details of a single company or set of companies. Moreover, the model
can detect anomalies in a quarterly format and will give managers and users of the report a more consistent update of the materiality of companies’ financial statements. Note also that each company’s quarter is ranked with anomaly ratings, which measures how anomalous the company’s indices are at a given point in time.

The insights from the anomaly ratings provided in Table 5 have the potential to play a significant role in understanding companies’ financial statements. A financial architecture that can manage OFSL risks will go a long way to protect the public interests and users of financial statements. Dishonest accounting practices could be of interest to some users in struggling companies and, as such, they may entertain the possibility that fraudulent accounting is permissible. However, company managers and regulators need to be cognizant of the fact that users want accurate and relevant information to make informed strategic decision; as such, anomaly detection models can prove useful in detecting anomalies in financial statements and offset deception, misreporting and the falsification of financial reports. In particular, the anomaly rating can be used to measure the level of accuracy of companies’ financials by users of financial statements (e.g., see Fanning and Cogger, 1998; Lyandres and Zhdanov, 2013). These findings provide meaningful insights to financial institutions when determining lending decisions and to investors when evaluating companies’ financials to make investment decisions (Bell & Carcello, 2000; Beneish et al., 2013; Behzadian and Izadi Nia, 2017; Lokanan, 2017).

At a more macro level, Vietnam has witnessed accelerated economic changes in its financial regulatory landscape in recent years (Narayan and Zheng, 2010; Phan et al., 2018). With such changes come pressures for companies to compete in one of the strongest growing markets in East Asia (World Bank, 2017; Phan et al., 2018). With such growth, the problem of financial statement manipulation has once again raised its ugly head in financial reporting (Hiep, 2017; Phan et al., 2018).

The results from this study also provide insights for government agencies to control and reduce the degree of financial statement fraud. As mentioned earlier, a very significant 23.51% of the testing dataset was shown to be anomalous. This result implies that regulators must focus more on enhancing transparency and compliance in financial reporting (Phan et al., 2018). In Vietnam, the Vietnamese Standards on Auditing (VSA) regulates the procedure for reviewing the quality of financial statements before they are released to the public. From both an audit and a user’s point of view, the models employed in this study can provide insights on best practices to improve the accuracy of audited financial statements. As management may want to manipulate earnings, the use of machine learning algorithms can be employed by regulators, preparers, auditors, and users to detect errors in financial reports and mitigate the prevalence of false representations cause by fraudulent reporting (Dbouk and Zaarour, 2017; Jan, 2018).

Due to the eclectic nature of anomaly detection in financial statements, a general model of outlier detection simply does not exist. As such, the regulatory apparatus must strive to identify models that can offer insights into different types of unethical (and at times fraudulent) behavior in financial reporting. In this regard, the paper advances research in anomaly detection by presenting a model that will assist individuals to analyze complex unethical behavior in accounting manipulation and, at the same time, offer deeper insights into fraud detection in financial statements. More importantly, and especially for the unranked companies, the paper can prove useful to examine managerial intent to act unethically (and sometimes fraudulently) while prioritizing and integrating interests of certain
users in the decision-making process.

7.1. Limitations of the Model

The machine learning algorithm employed in this paper suffers from several limitations. First, the missing values were treated in the same way in the financial statements. It was biased to do so because the missing values are the result of restriction to private data. In fact, the input data for this research are free and available, which mimics perfectly the insights that can be seen by ordinary stakeholders. This is very important as it not only intensifies human weakness when dealing with big data, but also reveals data imparity and scarcity in the Vietnamese market. Second, we had to employ statistical assumptions and treat the data as stemming from the normal distribution. The high dimension data require more advanced models, which are far beyond the scope of the data set we are working with (e.g., Fan and Palaniswami, 2000; Spathis, 2002; Shin et al., 2005). Third, the timeframe under study ranged from 2010 to 2016, which is a relatively short interval. Machine learning techniques require data acquired from a longer timeframe to ensure a better fitted model (Hoogs et al., 2007; Perols, 2011). Fourth, the threshold levels of anomalies are difficult to determine. As a result, the model is too sensitive to anomalies. With 23.51% of the companies' data points considered to be highly-anomalous, it can be said that they were not practical enough when compared to the historical record of the Vietnamese market.

7.2. Future Research

The present paper results in two significant findings which can serve as guidelines for further research on machine learning and detecting anomalies in financial statements. First, the data preprocessing methods used in this paper lay a good foundation for future research. Further research can examine how data pre-processing can transform raw data, which will be useful to users of financial information. Second, the result of the anomalous financial analysis can be contextualized in a more meaningful way than just finding out whether they are anomalous or not. One avenue is to pursue further research to see if companies that report anomalous data were cited for fraud or went on to commit fraud in the future.
References


Behzadian, F. and Izadi Nia, N. (2017) "An Investigation of Expectation Gap between Independent Auditors and Users from Auditing Services Related to the Quality of Auditing Services Based on Their Role and Professional Features", Asian Journal of Accounting Research, Vol. 2 Issue: 2, pp.36-47


No. 2, pp. 19–50.


About the Authors

Mark Lokanan is an Associate Professor in the Faculty of Management at Royal Roads University. He is a graduate from Simon Fraser University, Canada and is an expert in fraud, forensic and investigative accounting.

Vincent Tran is a final year business student at Royal Roads University

Vuong, Hoai Nam is a graduate student at Foreign Trade University - Faculty of International Economics