RUNNING HEAD: NEGATIVE NUMBERS

Negative Numbers in Simple Arithmetic

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Abstract

Are negative numbers processed differently than positive numbers in arithmetic problems?

In two experiments, adults ($N = 66$) solved *standard* addition and subtraction problems such as $3 + 4$ and $7 - 4$ and *recasted* versions that included explicit negative signs, that is, $3 - (-4)$, $7 + (-4)$, and $(-4) + 7$. Solution times on the recasted problems were slower than on standard problems, but the effect was much larger for addition than subtraction. The negative sign may prime subtraction in both kinds of recasted problems. Problem size effects were the same or smaller in recasted as compared to standard problems, suggesting that the recasted formats did not interfere with mental calculation. These results suggest that the underlying conceptual structure of the problem (i.e., addition vs. subtraction) is more important for solution processes than the presence of negative numbers.

Negative Numbers in Simple Arithmetic

Are negative numbers processed differently than positive numbers in arithmetic problems? Negative numbers are thought to be conceptually difficult for both children and adults (De Cruz, 2006). Although no one has addressed the question of whether negative numbers interfere with solving arithmetic problems, there is evidence suggesting that negative terms (e.g., $6 - x = 4$) might be more difficult for students to process than positive terms in algebraic equations (Peterson & Aller, 1971; Vlassis, 2004). We hypothesized that arithmetic problems with negative numbers may be processed differently because they usually require subtraction, rather than because of the negative number, per se. Subtraction is slower and more error-prone than addition (Campbell & Xue, 2001; LeFevre, DeStefano, Penner-Wilger, & Daley, 2006; Seyler, Kirk, & Ashcraft, 2003). On this view, the conceptual structure of the problem (i.e., the mental operation that is required) is the critical determinant of processing difficulty rather than the presence of a negative number.

In the present research, adults solved addition and subtraction problems in *standard* formats, that is $3 + 4$ and $7 - 4$, and as addition or subtraction of negative numbers in *recasted* formats, that is, $3 - (-4)$, $7 + (-4)$ and $(-4) + 7$. In the recasted formats, the conceptual structure of the problem did not change but the format was manipulated to explicitly isolate a negative number. Shaki and Petrusic (2005) outlined two hypotheses about how people process negative numbers. According to the magnitude-polarity hypothesis, the magnitude of the number and the polarity (i.e., positive vs. negative) are processed independently, as distinct features of the stimulus. In contrast, according to the number-line hypothesis, people represent negative and positive numbers along a number line, with negative numbers extending to infinity on the left and positive numbers extending to infinity on the right. Accordingly, the negative or positive status of the number is integral to mental processing. Ganor-Stern and Tzelgov (2008) labeled these two hypotheses the componential

and holistic views, respectively.

To explore the componential and holistic hypotheses, Shaki and Petrusic investigated adults' number comparison performance (i.e., choosing the larger or smaller number of a pair) in conditions where positive and negative number pairs (e.g., 4 vs. 3 and -4 vs. -3) were either presented in separate blocks, or were intermixed. When negative and positive number pairs were mixed, people appeared to use a number-line representation whereas when polarity was blocked, negative numbers were treated like positive numbers (i.e., only the magnitude was relevant). Thus, processing of negative and positive numbers in number comparison appears to depend on the task requirements and the context, rather than on the activation of a specific mental representation for negative numbers (see also Fischer, 2003; Fischer & Rottmann, 2005; Ganor-Stern & Tzelgov, 2008).

The most common use of negative signs in arithmetic is in the context of subtraction problems. Accordingly, subtraction problems such as $9 - 5$ or $14 - 9$ invoke slower processing and different solution approaches than complementary addition problems such as $5 + 4$ or $9 + 5$ (Campbell & Xue, 2001; LeFevre et al., 2006). Young adults often report using counting and other multi-step procedures on subtraction problems whereas retrieval from memory is used more frequently in addition (Campbell & Xue, 2001; LeFevre et al., 2006; Seyler et al., 2003). The response-time differences between operations in part reflect these differences in solution approaches (LeFevre et al., 2006; LeFevre, Sadesky, & Bisanz, 1996; Seyler et al., 2003). However, differences across operations could indicate that negative numbers are more difficult to process than positive numbers (cf. Vlassis, 2004).

Response latencies in arithmetic are also sensitive to problem size: problems composed of larger numbers (e.g., $9 + 7$; $16 - 7$) are solved more slowly and less accurately than problems composed of smaller numbers (e.g., $4 + 3$; $7 - 3$; Zbrodoff & Logan, 2005). The problem-size effect reflects, in part, differences in solvers' solution approaches. In

particular, the problem-size effect is greater when adults use procedural solutions (such as counting) than when they use memory retrieval to solve problems (LeFevre et al., 1996; Campbell & Fugelsang, 2001; Campbell, Parker, & Doetzel, 2004; Campbell & Penner-Wilger, 2006). Accordingly, the problem-size effect in subtraction is typically much larger than in addition, because solvers use procedural solutions more frequently (Campbell & Xue, 2001; LeFevre et al., 2006; Seyler et al., 2003). Furthermore, problem-size effects are generally larger in atypical formats than in typical or preferred formats, because atypical formats may cause participants to rely more heavily on procedural solutions (Campbell & Fugelsang, 2001; Campbell & Penner-Wilger, 2006; LeFevre, Shanahan, & DeStefano, 2004; Mauro, LeFevre, & Morris, 2003). Thus, we used the problem-size effect to evaluate whether negative numbers influence mental calculation processes in simple addition and subtraction.

The holistic and componential views of negative numbers predict different patterns of performance on simple arithmetic problems. Although both views predict main effects of format and interactions among the critical variables of operation, problem size, and format, the form of the expected interactions are distinct. If processing of negative numbers is holistic, negative numbers will disrupt mental calculation (because arithmetic facts are not stored in memory for equations with explicit negative numbers), and thus the holistic view predicts both a main effect of format and an interaction between format and problem size like that reported for other kinds of format manipulations (Campbell, 1994; Campbell & Fugelsang, 2001). For example, in a holistic solution to a problem like $(-4) + 7$, a solver may locate -4 on a mental number line and then move 7 steps in the positive direction to arrive at the answer $+3$ ¹. In contrast, for $7 - 4$, the solver may have stored the answer 3 and simply retrieve it from memory, without representing (-4) . On this view, processing negative numbers is equivalent to a manipulation of difficulty because negative numbers are assumed

to be harder for solvers to process than positive numbers. The holistic solutions for large equations with negative numbers such as $(-9) + 16$ would be even more affected because larger numerical distances would be challenging to represent and process using a mental number line. Accordingly, the holistic view predicts a three-way interaction among format, operation, and problem size such that the combined effects of problem size and format will affect the hardest problems the most (i.e., large recasted subtraction problems such as $16 + (-9)$)).

If processing of negative numbers is componential, however, then participants will also solve problems in recasted formats such as $5 - (-2)$ and $9 + (-4)$ more slowly than problems in standard formats because the recasted formats will require extra processing to extract the intended operation: $5 - (-2)$ becomes $5 + 2$ and $9 + (-4)$ becomes $9 - 4$ (i.e., a main effect of format). Importantly, however, subsequent mental calculations will be equivalent across standard and recasted formats, and thus format effects will be similar on small and on large problems (i.e., no interaction between format and problem size). Nevertheless, format and *operation* may interact, if extra processing of the recasted format takes longer for one operation than for the other. Based on our initial speculation that processing of negative numbers in arithmetic is essentially a subtraction operation, we predicted that recasted formats will affect addition problems more than subtraction problems. On this view, we expect a three-way interaction among format, problem size, and operation such that the format effects will be smallest for the ostensibly hardest problems (i.e., recasted large subtraction). Such an interaction is counter-intuitive if negative numbers are considered to be more difficult than positive numbers (e.g., Vlassis, 2004), but existing research on negative numbers in comparison tasks favors a componential view (Ganor-Stern & Tzelgov, 2008; Shaki & Petrusic, 2005).

Experiment 1

Method

Participants

Thirty-five undergraduate students received credit in their introductory psychology class for participating in this experiment. However, three individuals did not know how to solve the recasted problems, and their data were not analyzed. The 32 remaining participants (16 males and 16 females) ranged in age from 17 to 43 years ($Mdn = 22$). Eleven participants had received all of their education in Canada. The others had received their education (prior to university) in a country other than Canada. Participants were native speakers of English ($n = 16$), Chinese ($n = 12$), Russian, Bulgarian, Tamil, or Urdu (one of each).

Materials

Addition problems. The problem set was composed of the 81 possible combinations of first operands 1 to 9 and second operands 1 to 9 in the format $a + b$, and 81 problems in the format $a - (-b)$. Two lists of problems were created, such that one of the complementary problems $a + b$ was in one list and $b + a$ was in the other list (hence 36 per list). All nine ties (i.e., $a = b$) were included in each list, for a total of 45 per list. Small addition problems comprised 25 combinations of operands in each list such that answers (sums) were smaller than 11 (e.g., $5 + 1 = 6$; $5 - (-2) = 7$). Large addition problems comprised 20 combinations of operands in each list such that answers were greater than 10 (e.g., $8 + 9 = 17$; $7 - (-8) = 15$).

Subtraction problems. The problem set was composed of 81 possible combinations of first operands 18 to 2 and second operands 1 to 9 in the form $c - b$; and 81 problems of the form $c + (-b)$. The a and b values corresponded to those used in the addition problems. Thus, all 9 tie combinations were included (e.g., $18 - 9$) and one of each complementary problem (i.e., $c - b$ and $c - a$). Small subtraction problems comprised 25 combinations of first operands and second operands such that $0 < c < 11$ (e.g., $6 - 1 = 5$; $5 + (-2) = 3$). Large subtraction

problems comprised 20 combinations of first operands and second operands such that $c > 10$ (e.g., $17 - 9 = 8$; $14 + (-8) = 6$).

Each participant saw a pseudo-randomized order of one problem list. Problems were ordered with the constraints that no first or second operand, sum, or remainder, were repeated on consecutive trials.

Other measures. Participants completed the addition and subtraction-multiplication subtests of the French Kit (French, Ekstrom, & Price, 1963). Each subtest of this paper-and-pencil task consists of two pages of multi-digit arithmetic problems. The participants were instructed to solve the problems as quickly and accurately as possible and were given 2 min per page. Arithmetic fluency was measured as the total number of correct solutions on both tests combined and reflected an individual's ability to quickly and accurately execute simple arithmetic procedures on multi-digit problems. Participants also completed two questionnaires; a brief one regarding the strategies they used to solve both standard and recasted addition and subtraction problems and a detailed questionnaire regarding educational background, past experience with arithmetic, self-rated arithmetic skill, and use of arithmetic strategies.

Procedure

Each participant was individually tested in a single session lasting not more than 60 min. Arithmetic trials were administered first, followed by the paper-and-pencil arithmetic test and the background and strategies questionnaires. Before the session began, the participant was told that the purpose of the study was to investigate differences in processing simple addition and subtraction problems, some with negative numbers. Each session was composed of two blocks: one block consisted of standard addition (e.g., $3 + 4$) and recasted subtraction problems (e.g., $7 + (-4)$); and one block of standard subtraction (e.g., $5 - 2$) and recasted addition problems (i.e., $5 - (-2)$). The order in which participants saw the two blocks

was counterbalanced across participants.

The arithmetic problems were presented horizontally on a monochrome monitor connected to a computer equipped with a Pentium III processor. A Linux program written in C controlled the computer. Timing was accurate to the nearest ms. Stimuli were presented in white on a black background. After viewing instructions for the block of trials on the monitor, participants initiated each block of problems by saying, “go”. For each trial, an asterisk was presented in the centre of the screen, and flashed twice, with the problem appearing on what would have been the third flash, one second from the start of the trial. The operation sign appeared in the same position as the asterisk. The problem remained on the screen until the participant made a verbal response, or until a 15 s deadline was reached. The experimenter recorded the response and the next trial began with the presentation of the asterisk. Trials were scored as invalid by the experimenter if (a) the microphone was triggered by a participant’s extraneous vocalization (e.g., a cough), (b) the participant’s response was too quiet to trigger the microphone, or (c) the participant was unable to produce a response.

Results and Discussion

Questionnaires and Arithmetic Fluency

Participants described a variety of procedures for solving simple arithmetic problems, including retrieval from memory (i.e., ‘knowing the answer’), decomposition (e.g., $15 - 8 = [15 - 5] - 3$), counting (e.g., solving $9 + 2$ by counting up 10, 11), and knowledge of sign rules (i.e., negative plus negative = positive; positive plus negative = negative). For arithmetic problems with negative numbers, most participants reported using their knowledge of sign rules.

The scores on the arithmetic fluency test for these participants ranged from 33 to 129, with a mean of 84 ($SD = 24.5$). The mean for samples of Canadian-educated undergraduates

on this measure in recent years has ranged from 60 to 70 ($SD = 25$; LeFevre et al., 2008).

Hence, the participants in the current experiment had a higher level of skill than anticipated.

This was in part due to the substantial percentage of Asian-educated students (12/32), whose mean score was 101.

Arithmetic Problems

Each participant solved 180 arithmetic problems for a total of 5760 trials across the 32 participants. Of these, 218 trials were errors and 63 were invalid, due to inadvertent voice key triggers. This error rate (3.8%) is similar to those reported in other production experiments (i.e., 2% in Miller, Perlmuter, & Keating, 1984; 3% in Geary & Wiley, 1991). Because the error rate was low and uniform across participants, errors were not analyzed further.

Latencies faster than 400 ms ($n = 66$) and slower than 6000 ms ($n = 17$) were also excluded from the analysis. For the remaining 5431 trials, median latencies were calculated for each participant in each condition and analyzed in a 2 (problem size: small, large) $\times$ 2 (problem format: standard, recasted) $\times$ 2 (operation: addition, subtraction) repeated measures ANOVA. Reported results were significant at $p < .01$, unless otherwise indicated.

Participants solved large problems (1363 ms) more slowly than small problems (977 ms), $F(1, 31) = 47.03$, $MSE = 202959$, $\eta_p^2 = .60$. Latencies varied with operation such that addition problems were solved more quickly than subtraction problems (1086 vs. 1254 ms), $F(1, 31) = 37.64$, $MSE = 48363$, $\eta_p^2 = .55$. Furthermore, operation and size interacted such that the problem-size effect was greater on subtraction (488 ms) than on addition problems (284 ms), $F(1, 31) = 20.93$, $MSE = 31643$, $\eta_p^2 = .40$. These effects are consistent with existing literature.

Of most interest was the effect of format. Participants solved problems in standard formats more quickly than in recasted formats (1094 vs. 1246 ms), $F(1, 31) = 106.15$, $MSE = 13795$, $\eta_p^2 = .77$. Furthermore, format and operation interacted, $F(1, 31) = 13.91$, $MSE =$

18651, $\eta_p^2 = .31$. The difference between standard and recasted problems was larger in addition than in subtraction, as shown in Figure 1. There was no significant interaction between format and problem size, $F(1,31) = 1.09$, $MSE = 9022$, $\eta_p^2 = .03$. This pattern of results supports the componential hypothesis.

< Insert Figure 1 about here >

To better understand whether format influenced mental calculation, we examined the three-way interaction of problem size, problem format, and operation, $F(1, 31) = 3.36$, $MSE = 7587$, $p = .076$, $\eta_p^2 = .10$. As shown in Figure 2 (top left panel), for addition there was an equivalent format cost on both small and large problems (207 vs. 222 ms) whereas for subtraction, differences between standard and recasted formats were 120 and 55 ms, for small and large problems respectively. In separate analyses by operation, the interaction between format and problem size was not significant for addition, $F(1,31) < 1.0$, $\eta_p^2 = .01$. For subtraction, although the interaction was not significant, the effect size was modest, $F(1, 31) = 2.72$, $p = .109$, $\eta_p^2 = .08$. This trend towards smaller format costs for the hardest problems (i.e., large subtraction) is the opposite of what would be predicted in the holistic view.

<Insert Figure 2 about here>

Summary

The results of Experiment 1 are consistent with the hypothesis that participants initially process recasted problems to determine the required operation, and subsequently solve the extracted arithmetic problem. The interaction between format and operation, such that more processing time is required on recasted addition problems than on recasted subtraction problems, also supports the view that this processing does not involve mental calculation. The mental calculation required for subtraction is more complex and difficult than for addition (e.g., LeFevre et al., 2006) and thus if the recasting influenced mental calculation, we would expect a larger effect on subtraction. Instead, it seems possible that the

presence of a negative sign primes the subtraction operation and thus subtraction problems are facilitated, whereas addition problems that involve negative signs are slowed.

Furthermore, the findings that (a) format affects small and large problems equivalently in addition and (b) format affected large subtraction less than small subtraction (thus opposite to what might be expected in the holistic view), further supports the conclusion that the presence of negative signs in arithmetic problems does not influence mental calculation directly. Although the results are consistent with the predictions of the componential view, they are nevertheless somewhat counterintuitive. Hence, it was important to replicate the findings in a second experiment. Thus, the first goal of Experiment 2 was to replicate the pattern of findings and thus provide another test of the componential hypothesis. A second, related goal was to extend the results to a somewhat different version of recasted subtraction problems, $(-b) + c$, that was less easily interpreted as a subtraction problem than $c + (-b)$. The latter format may not have adequately tested the holistic view because the format was very similar to that of standard subtraction. Third, in Experiment 2, problems were not blocked by operation sign as in Experiment 1. Research with negative numbers in comparison tasks suggests that participants may process negative numbers componentially in blocked designs but holistically when positive and negative numbers were mixed together (e.g., Shaki & Petrusic, 2005). Hence, holistic processing might be more likely when the problems are not blocked.

Experiment 2

Method

Participants

Thirty-four adults (15 females) participated in this experiment and received credit towards their introductory psychology course. Median age was 19 years. Twenty-eight of the participants had received their high school education in Canada, two in China, and the

remainder in other countries. English was the first language of 25 participants, with the others reporting Chinese ($n=3$), Farsi ($n=4$), and one each Bulgarian and Serbian.

Materials

In addition to the standard and recasted problems used in Experiment 1, participants also saw problems in a third recasted format: $(-b) + c$. In each of the five problem types, participants solved 54 problems, rather than 45 as in Experiment, because each of the 9 tie problems were presented twice instead of once. Thus, each participant solved a total of 270 problems. In contrast to Experiment 1, problems were not blocked by operation sign. Instead, order of the problems was determined randomly for each participant.

Procedure

The procedure was identical to that in Experiment 1, with the difference that the arithmetic problems were presented using Superlab on an iMac.

Results and Discussion

Arithmetic Fluency

Compared to Experiment 1, more of the participants in Experiment 2 received their education in Canada (82% vs. 34%, respectively). Participants in Experiment 2 also showed a wider range of performance on the arithmetic fluency test, from 14 to 171, and a lower average score ($M = 67.3$, $SD = 31.6$). The lower overall level of arithmetic fluency is reflected in slower response times on the arithmetic problems (as shown in Figures 1 and 2). However, as discussed below, the sample characteristics did not result in a different pattern of performance than was found in Experiment 1.

Responses to the questions about how participants solved the recasted problems were similar to those in Experiment 1. Most participants described a form of the ‘sign rule’ on recasted addition problems, for example. Most reported a variety of solution procedures

including memory retrieval. On the recasted subtraction problems, participants reported ignoring the brackets, or that they saw the problem as subtraction automatically.

Arithmetic Problems

Each participant solved 270 arithmetic problems for a total of 9180 trials across the 34 participants. Of these, 564 trials (6.1%) were errors and 105 were invalid (1.1%), due to inadvertent voice key triggers or experimenter errors. Median latencies on correctly solved trials and percentage correct were analyzed in 2(format: standard, recasted) x 2(operation: addition, subtraction) x 2(problem size: small, large) repeated measures ANOVAs. Reported results were significant at $p < .01$, unless otherwise indicated.

Accuracy was analyzed in this experiment because it was lower overall than in Experiment 1 and because some participants made a substantial number of errors. Participants were more accurate on small than on large problems (95 vs. 90%), on addition than on subtraction problems (94 vs. 91%), and on standard than on recasted problems (94 vs. 91%), $F_s(1,33) > 7$. None of the interactions were significant for accuracy, however, and thus the latency analyses were not compromised by tradeoffs with accuracy.

For latencies, participants responded more quickly on standard than on recasted problems (1184 vs. 1594 ms), on addition than subtraction problems (1270 vs. 1419 ms), and on small than large problems (1141 vs. 1548 ms), $F_s(1,33) > 25$, $\eta_p^2 > .44$. Operation interacted with problem size, $F(1,33) = 8.51$, $MSE = 26,380$, $\eta_p^2 = .20$. The problem size effect was greater for subtraction (1187 vs. 1652 ms) than for addition (1095 vs. 1444 ms). These effects all replicate those reported in Experiment 1.

As in Experiment 1, format and operation interacted, $F(1,33) = 15.53$, $MSE = 20,838$, $\eta_p^2 = .32$. The format effect was larger for addition than for subtraction as shown in Figure 1 (right panel). Negative signs may signal ‘subtraction’ and thus cause greater disruption in

processing for addition than subtraction problems. There was no significant interaction between format and problem size, $F(1,33) = 1.43, p = .241, \eta_p^2 = .04$.

The three-way interaction of format, operation, and problem size was significant, as shown in Figure 2, $F(1,33) = 4.86, MSE = 17,307, p < .05, \eta_p^2 = .13$. The difference between recasted and standard addition problems was very similar for small and large addition problems (381 vs. 397 ms), somewhat less on small subtraction problems (313 ms), and substantially less on large subtraction problems (188 ms). This is the same pattern of format costs shown by the participants in Experiment 1. Separate analysis of the format by problem size effects for each operation supported the trend observed in Experiment 1. For addition, the interaction was not significant, $F(1,33) < 1.0, \eta_p^2 = .005$. The format effect was equivalent for small and large problems. For subtraction, however, there was a trend towards a greater format effect on small than on larger problems, $F(1,31) = 3.45, p = .071, \eta_p^2 = .10$. Hence, the hardest problems were least affected by the format manipulation, a result that is not consistent with the predictions of the holistic view.

An additional recasted format was included in Experiment 2, $(-b) + c$. The hypothesis was that, if negatively signed numbers have some special status, then this format might be processed differently than the conceptually identical problem $c + (-b)$ because the latter might be encoded directly as a subtraction problem $c - b$, without considering b as negative. Latencies on these two subtraction formats were analyzed in a $2(\text{format: } c + (-b), (-b) + c) \times 2(\text{problem size: small, large})$ repeated measures ANOVA. According to this analysis, latencies on the two recasted formats were indistinguishable (1545 vs. 1542 ms), $F(1,33) < 1$. As expected, latencies varied with problem size (1323 vs. 1764 ms), $F(1,33) = 41.45, MSE = 159631, \eta_p^2 = .56$, but format and problem size did not interact, $F(1,33) = 1.43, \eta_p^2 = .04$. Thus, at least within this experimental context, the negative sign was not processed differently across these two versions of the recasted subtraction problems.

General Discussion

Are negative numbers processed differently than positive numbers in arithmetic problems? Participants solved addition and subtraction problems in standard (e.g., $3 + 4$; $7 - 4$) and recasted formats (e.g., $3 - (-4)$ and $7 + (-4)$). Participants solved addition problems faster than subtraction problems. They also solved standard problems faster than recasted problems. The results suggest that the presence of a negative number, per se, was not the main determinant of calculation processes. The absence of interactions between format and problem size in each experiment supported the componential view in that small and large problems were equally affected by format. Further, patterns of interaction between operation and format supported the notion that in the context of arithmetic, negative signs are interpreted as indicating subtraction. Specifically, addition problems were slowed more by the recasted format than were subtraction problems in both experiments. Because the presence of a negative sign in arithmetic problems usually signals “subtract”, it is likely that the operation of subtraction is primed in a problem such as $6 - (-4)$ and deliberate strategic processing is required to overcome this priming, apply the appropriate sign rule, and then solve the embedded addition problem. In contrast, for all of the subtraction formats; $6 - 4$, $6 + (-4)$, and $(-4) + 6$, the presence of a negative sign activates or signals the appropriate operation. Thus, application of a rule, or restructuring the problem as subtraction, may be facilitated as reflected in the smaller effect of recasting on subtraction than addition.

The calculation processes for problems in recasted formats (addition or subtraction) did not appear to be disrupted by the additional strategic demands to select the appropriate operation. On small (easier) and large (harder) addition problems, participants showed parallel results across standard and recasted formats. Harder subtraction problems, if anything, seemed to benefit from the recasted format in that large problems were less affected by format than were smaller problems. In contrast, for other unusual formats such as word

problems (e.g., five + eight), format costs are greater on large than on small problems. In part, these format costs have been attributed to reduced use of retrieval for word problems (Campbell & Penner-Wilger, 2006) and thus the format costs are interpreted as affecting mental calculation. Our results suggest that the present format manipulation (i.e., negative signs) did not influence mental calculation. Instead, the results are consistent with the hypothesis that in arithmetic problems, negative signs are processed in an encoding stage that precedes calculation processes.

In summary, the present results suggest that processing of negative numbers in simple arithmetic is componential (Ganor-Stern & Tzelgov, 2008; Shaki & Petrusic, 2005).

Negative signs were processed such that participants first determined which operation was required, and then implemented it. Further calculation processes depended on the operation that was required (i.e., addition or subtraction) rather than on the presence of a negative number. Thus, the presence of negative numbers did not disrupt mental arithmetic.

References

- Campbell, J. I. D., & Fugelsang, J. (2001). Strategy choice for arithmetic verification: Effects of numerical surface form. *Cognition*, 80, B21-30.
- Campbell, J. I. D., Parker, H. R., & Doetzel, N. L. (2004). Interactive effects of numerical surface form and operand parity in cognitive arithmetic. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, 30, 51-64.
- Campbell, J. I. D., & Penner-Wilger, M. (2006). Calculation latency: The μ of memory and the τ of transformation. *Memory & Cognition*, 34, 217-226.
- Campbell, J. I. D., & Xue, Q. (2001). Cognitive arithmetic across cultures. *Journal of Experimental Psychology: General*, 130, 229-315.
- De Cruz, H. (2006). Why are some numerical concepts more successful than others? An evolutionary perspective on the evolution of number concepts. *Evolution and Human Behavior*, 27, 306-323.
- Fischer, M. H. (2003). Cognitive representation of negative numbers. *Psychological Science*, 14, 278-282.
- Fischer, M. H., & Rottmann, J. (2005). Do negative numbers have a place on the mental number line? *Psychological Science*, 47, 22-32.
- French, J. W., Ekstrom, R. B., & Price, I. A. (1963). *Kit of reference tests for cognitive factors*. Princeton, NJ: Educational Testing Service.
- Ganor-Stern, D., & Tzelgov, J. Negative numbers are generated in the mind. *Experimental Psychology*, 55, 157-163.
- Geary, D., & Wiley, J. (1991). Cognitive addition: Strategy choice and speed-of-processing differences in young and elderly adults. *Psychology and Aging*, 6, 474-483.
- LeFevre, J., Shanahan, T., & DeStefano, D. (2004). The tie effect in simple arithmetic: An access-based account. *Memory & Cognition*, 32, 1019-1031.

- LeFevre, J., DeStefano, D., Penner-Wilger, M., & Daley, K. E. (2006). Selection of procedures in mental subtraction. *Canadian Journal of Experimental Psychology*, 60, 209-220.
- LeFevre, J., Sadesky, G. S., & Bisanz, J. (1996). Selection of procedures in mental addition: Reassessing the problem size effect in adults. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, 22, 216-230.
- Mauro, D., LeFevre, J., & Morris, J. (2003). Effects of problem format on division and multiplication performance: Division facts are mediated by multiplication-based representations. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, 29, 163-170.
- Miller, K. F., Perlmuter, M., & Keating, D. (1984). Cognitive arithmetic: Comparison of operations. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 10, 46-60.
- Peterson, M. J., & Aller, S. (1971). Arithmetic problem solving. *Journal of Experimental Psychology*, 91, 93-97.
- Seyler, D. J., Kirk, E. P., & Ashcraft, M. H. (2003). Elementary subtraction. *Journal of Experimental Psychology: Learning, Memory, & Cognition*, 29, 1339-1352.
- Shaki, S., & Petrusic, W. M. (2005). On the mental representation of negative numbers: Context dependent SNARC effects with comparative judgments. *Psychonomic Bulletin & Review*, 12, 931-937.
- Vlassis, J. (2004). Making sense of the minus sign or becoming flexible in 'negativity'. *Learning and Instruction*, 14, 469-484.
- Zbrodoff, N. J., & Logan, G. D. (2005). What everyone finds: The problem-size effect. In J. I. D. Campbell (Ed.), *Handbook of Mathematical Cognition* (pp. 331-346). New York: Taylor & Francis.

Footnotes

1. We thank an anonymous reviewer for this useful example.

Figure Captions

Figure 1. Format by operation interactions in Experiments 1 and 2. Whiskers represent the 95% confidence intervals calculated using the *MSE* from the appropriate interactions.

Figure 2. Means of median solution times for the interactions of format, problem size, and operation in each experiment. Whiskers represent the 95% confidence intervals calculated using the *MSE* values from the three-way interactions.