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Optimization of array configuration [43.30.Y], 43.60.Gk]

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The purpose of this study was to determine the optimal configuration of 2-, 3-, and 4-element hydrophone arrays. The noise field was taken to be isotropic and the arrays were constrained to lie in a horizontal plane. It was determined that the optimal configuration is a linear array though, as is the case of the 4-element array, not necessarily uniformly spaced. In addition, near-optimal configurations, which are nonlinear, were found whose beam patterns and frequency dependence may be preferable in specific situations.

INTRODUCTION

The use of an array of hydrophones to improve the output signal-to-noise ratio is widespread. As shown in Fig. 1, the individual sensor outputs are multiplied by weights and are then summed to form the array output. The array gain is dependent on both the hydrophone positions, through the cross-correlation coefficients, and the applied weights. Optimization of array gain generally involves optimizing the weights for a given array configuration. The problem of determining sensor weights is the same as designing a data window for spectral estimation.¹

FIG. 1. General method of processing hydrophone array inputs $\bar{X}$.

Optimization of array gain can also be considered in terms of optimizing the array configuration for a given set of weights. Horton² considered the dependence of array gain on the spacing of an equally spaced linear array. This paper will concern itself with the more general case where the constraints of both equal spacing and linearity are removed. The problem that will be addressed will be to determine the configuration of 2-, 3-, and 4-element arrays that will provide the maximum array gain or output signal-to-noise ratio, in a given look direction and at a given signal frequency. The noise field will be taken to be isotropic, the array will be constrained to lie in a horizontal plane, and the weights $\bar{W} = \bar{I}$.

I. DEPENDENCE OF ARRAY GAIN ON ARRAY CONFIGURATION

The general expression for the array gain (AG) of N sensors is³

$$AG = 10 \log \left(\frac{\sum_{ij} W_i W_j (\rho_s)_{ij}}{\sum_{ij} W_i W_j (\rho_n)_{ij}} \right), \quad (1)$$

with $(\rho)_{ij}$ representing the normalized cross-correlation coefficients, after phase shifts have been added for steering, for the signal and noise inputs, respectively. For $\bar{W} = \bar{I}$ and a perfectly coherent unidirectional signal $\sum_{ij} W_i W_j (\rho_s)_{ij} = N^2$. For an array constrained to the x - y plane and with a look direction of 90° , with respect to the x axis the cross-correlation coefficient for a spherically symmetric noise field is given by

$$(\rho_n)_{ij} = (\sin 2\pi d_{ij} / 2\pi d_{ij}) \cos(2\pi d_{ij} \sin \theta_{ij}), \quad (2)$$

with d_{ij} and θ_{ij} representing the separation of sensors i and j in wavelengths and the angle between the axis connecting the two sensors and the x axis.

The problem of maximizing array gain thus reduces, for this specific problem, to minimizing the following summation:

$$N(d_{ij}, \theta_{ij}) = \sum_{ij} \frac{\sin 2\pi d_{ij}}{2\pi d_{ij}} \cos(2\pi d_{ij} \sin \theta_{ij}). \quad (3)$$

This is a function of $2(N-1)$ variables and, for $N > 2$, can be solved using a search algorithm.

II. RESULTS

A. Two sensors

For two sensors, the minima of Eq. (3) can be found analytically. With one hydrophone taken to be at the origin, the position of the second sensor that minimizes $N(d_{ij}, \theta_{ij})$ and thus maximizes the array gain is 0.715λ for broadside steering, and half this spacing for endfire steering. The array gain realized is 4.07 dB. The dependence of array gain on the position of the second sensor is shown in Fig. 2. This figure shows that the maxima are relatively broad with errors of 10% in the positioning of the second sensor having little effect on the realizable array gain.

The frequency dependence of the array gain for both endfire and broadside steering is the same with improved array gain with respect to 3.01 dB, the gain obtainable in uncorrelated noise, over approximately a 1-oct band. The dependence of the array gain on look direction for the two optimal configurations is different. Defining the view angle, as with frequency range, to be the region between the 3.01-dB points, it is found that for endfire steering the view angle is approximately 92° while for broadside steering the range is approximately 41° .

B. Three sensors

For $N > 2$, the minima of Eq. (3) were found by using a search algorithm. In the case of a 3-element array, two optimal configurations, corresponding to broadside and endfire steering, and one near-optimal configuration were found; these are shown in Fig. 3. The array gains obtainable in the look direction, 90° with respect to the x axis, for the three configurations are 6.28 dB for the broadside and endfire configurations and 6.22 dB for the triangular configuration. These compare with an array gain of 4.77 dB for a 3-element array in uncorrelated noise.

With respect to frequency dependence, the three configurations have improved array gain, with respect to the gain obtainable in uncorrelated noise, over approximately a 1-oct band. However, there are differences between the three configurations with regard to the dependence of array gain on look direction. The view angles for the three possible configurations, using the 4.77-dB points as upper and lower limits, are (a) endfire— 78° , (b) broadside— 34° , and (c) triangular— 54° .

C. Four sensors

With four sensors, two optimal configurations corresponding to broadside and endfire steering and four near-optimal configurations were found. The two optimal and one of the near-optimal configurations are shown in Fig. 4. The array gain obtainable in the look direction for broadside and endfire steering is 7.78 dB and for the rectangular configuration is

FIG. 2. Dependence of array gain on the position of the second sensor of a 2-element array. Contour lines are every 0.5 dB with $\times$ indicating optimum positions, λ_0 is the wavelength of the received signal, and (x,y) is the position of the second sensor.

FIG. 3. Optimal configurations for a 3-element array in an isotropic noise field.

7.77 dB. As shown in Fig. 4, the elements are not equally spaced for the broadside and endfire configurations. In the case of broadside steering, constraining the sensors to be equally spaced leads to an optimal spacing of 0.828λ and an array gain of 7.77 dB. These array gains are to be compared with 6.02 dB, the array gain obtained from a 4-sensor array in uncorrelated noise. As stated, other near-optimal configurations exist; however, their performance with respect to their dependence on frequency and look direction lies within the bounds of the above three configurations.

The dependence of the array gain on frequency for the three orientations is similar with improved array gain, with respect to 6.02 dB, over approximately a 1-oct band. The view angles for the three configurations, as labeled in Fig. 4, are (a) endfire— 73° , (b) broadside— 30° , and (c) rectangular— 58° . Thus there are significant differences with regard to the three orientations in this respect.

III. DISCUSSION

The scenario in which the above-discussed optimization procedure should be implemented is that of a limited view angle. Such situations occur in channels and inlets or in areas where the bathymetry conditions limit the view. It was determined that, in the case of isotropic noise, the array gain in a limited view angle could be significantly increased by optimizing the sensor positions. It was found that the optimal array configuration for an isotropic noise field is a linear array, either broadside or endfire steered. However, other near-optimal configurations exist whose beam patterns or frequency dependence may be preferable in specific situations. It was also found that, in the case of a 4-element array, the elements are not uniformly spaced for the optimal configuration, although the improvement over an equally spaced array is not significant. For signal and noise fields other than those considered in this paper, expressions for the actual signal and noise cross-correlation coefficients, which may be experimentally determined, need to be substituted into Eq. (1).

The optimization of sensor position was done for the case $\bar{W} = \bar{I}$. The

FIG. 4. Optimal configurations for a 4-element array in an isotropic noise field.

generalized problem would be to optimize both sensor positions and the applied weights. For an array confined to two dimensions and complex weights, this will involve optimizing Eq. (1) which will be a function of $4(N - 1)$ variables. In the case of adaptive signal processing, the optimum sensor positions would first be determined, for a given $\bar{W}_{\text{initial}}$, and then the weights allowed to adapt to changing conditions.

¹F. J. Harris, "Windows, Harmonic Analysis, and the Discrete Fourier Transform," Naval Undersea Center, Rep. TP-532, pp. 1-67 (1976).

²J. W. Horton, *Fundamentals of Sonar* (U.S. Naval Institute, Annapolis, MD, 1957), p. 194.

³R. J. Urick, *Principles of Underwater Sound* (McGraw-Hill, New York, 1975), p. 34.