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Modelling Nonlinearities in commodity prices using smooth transition regression models with external transition variables

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price captured the late ones (between 1980 and 2007). This result motivates the use of *external* threshold variables in regime switching models in general and, in particular, the use of inflation and oil price in the STR model when applied to an index of commodity prices.

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1 Introduction

During the 1970's, the increase in the price of oil and many other commodities, such as sugar, cotton, gold and silver, dominated macroeconomic discussions. In the late 1980's and early 1990's, there was a general decline in commodity prices and, consequently, the topic fell out of favor. Currently, the surge in the price of oil and many other commodities, both in nominal and real terms, drew back attention to the issue.

Studying the dynamic behavior of commodity prices over time and whether or not they are driven by common transition variable(s) are the main objectives of this paper. To this end, technical issues are considered in detail. In particular, the paper attempts to answer questions like: How are the dynamics of commodity prices to be modelled? Is (are) there common driving variable(s) responsible for such changing dynamics? And if these variables exist, what is the economic rationale behind choosing them? Why treating an index of commodity prices is different than treating individual commodities? And why a

border price classification is crucial to identify the potential driving variables for *individual* commodities?

The issue of commodity price formation has been studied extensively in the literature. Early theoretical models originate from Gustafson's (1958) work on the theory of competitive storage and the work of Muth (1961), who introduced the rational expectations assumption in a model of commodity price formation. Both contributions formed the basic model of commodity price formation. Extensions (in different directions) to this basic model can be seen from the work of Samuelson (1971), Danthine (1977), Williams and Wright (1991), and Deaton and Laroque (1992). The previously mentioned attempts, elegant as they were, did not succeed in capturing entirely the dynamic behavior of commodity prices. The general conclusion is that commodity prices are nonlinear and this nonlinearity is attributed to the speculative behavior of agents for holding stocks or to unobserved demand and supply shocks.

This paper follows a different approach in modelling the dynamics of commodity prices. The approach is empirical in nature and is motivated by the fact that commodity prices tend to move together in groups that can be classified according to the recorded *border price* of the commodity under consideration. The issue of co-movements of commodity prices refers to the tendency of some commodities to move together in response to a *common* macroeconomic variable. This paper seeks to explain this fact by, first, classifying various commodities according to their border price (an issue that has been ignored in previous studies), and then by attempting to find the best common macroeconomic variable that is responsible for this co-movement. Figure 1 illustrates this idea. The commodity price index used in this research is the Grilli and Yang's (1988) commodity price index, which is a composite index of 24 commodity prices. The individual price processes are classified into different groups based on the

recorded border price.¹ The time series in each group may be thought of as remaining in a given regime until pushed to a new regime by a shock, a series of shocks, or by a *common* driving (transition) variable. One can interpret such dynamics as a switching behavior between multiple equilibria. Finding those common driving variables and modelling this switching behavior are the main objectives of this paper.

The data set published by Grilli and Yang (1988), which is used in this paper, was originally developed and used by the authors to study the long-run behavior of the net barter terms of trade series or, in other words, to test the Prebisch (1950) and Singer (1950) hypothesis (PS hypothesis) of a secular decline in the price of primary commodities relative to the price of manufactures. To this end, the authors developed a commodity price index that consists of 24 primary commodity prices and known as the Grilli and Yang Commodity Price Index (GYCPI). The authors deflated the GYCPI by an index of manufactured goods' unit values² (MUV) and fitted a log-linear time trend model to the ratio GYCPI/MUV as well as to the individual commodities forming the index. They found a significant downward trend in the net barter terms of trade; that is, the estimated coefficient obtained from regressing the logarithm of the ratio GYCPI/MUV (denoted y_t in the text) on a linear time trend has a negative sign. Applying this trend stationary (TS) model to y_t , the authors found support for

¹Border prices, also referred to as International Commercial Terms (Incoterms), are the terms of selling that define the obligations of the trading parties engaged in trading contracts. Among all border prices, two are used the most: Free on board (FOB) prices and cost insurance and freight (CIF) prices. A trading contract effected on a FOB basis implies that the exporter (shipper) bears all the risks and costs of transporting the cargo from the point of origin (e.g., the exporter's factory) to the port of export in the country of origin (exit point of the exporting country). The importer (consignee) bears all the risks and costs of the cargo from that point up to delivery to final destination. A CIF price, on the other hand, is a FOB price plus insurance cost plus ocean freight cost; that is, under a CIF contract, the exporter, in addition to the insurance, bears all risks and costs of transporting the cargo from the point of origin to the port of discharge (entry point of the importing country).

²The MUV is a trade-weighted index of the five major developed countries' (France, Germany, Japan, United Kingdom, and United States) exports of manufactured commodities to developing countries.

Figure 1: Border price classification of individual commodities in the Grilli and Yang (1988) data set.

the PS hypothesis.

Using the data set published by Grilli and Yang, the PS hypothesis has been re-assessed by a number of authors using different econometric time series models; Grilli and Yang (1988), Helg (1991), Ardeni and Wright (1992), and Bleaney and Greenaway (1993) fitted a TS model; Cuddington and Urzua (1989) and Newbold and Vougas (1996) fitted both TS and DS models; Von Hagen (1989) used the bivariate approach and tested the hypothesis that the price series are cointegrated. Lutz (1999) combined the TS, DS, and the cointegration relation into an encompassing first-order distributed lag model with its error-correction equivalent. Persson and Teräsvirta (2003) assumed, as a starting point to the analysis, that the series y_t are stationary. Using the extended series of Lutz (1999), the authors, unlike all the previous studies, considered the hypothesis that the series might be nonlinear. In particular, they tested the linearity hypothesis in y_t against a parametric nonlinear model (the smooth transition autoregressive model) and could not reject the nonlinearity in the price series given the stationarity assumption. If the price series is nonlinear and stationary, the resulting mean reversion behavior will contradict the PS hypothesis. Therefore, the authors reached the same conclusion as of Newbold and Vougas (1996) that the findings vary according to the starting point of the analysis.

In this paper, we follow the same path as of Persson and Teräsvirta (2003) that the GYCPI as well as the individual price processes forming the index are nonlinear. We test the nonlinearity hypothesis and we model the dynamics of the price processes using regime switching models. In particular, we employ the smooth transition regression (STR) model pioneered by Granger and Teräsvirta (1993) and Teräsvirta (1994). STR models have been used extensively in the regime switching literature (see, van Dijk, Teräsvirta, and Franses

(2002) for a recent survey). In addition to their popularity, STR models possess some appealing features. They are based on a three-stage modelling procedure starting from a specification stage, estimation, and finally an evaluation stage. Also, as opposed to other regime switching models (e.g. pure threshold models and Markov switching models), STR models have the flexibility of describing processes that can move from one regime to the other such that the transition is smooth. Threshold models are best suited here because, unlike other nonlinear models (Markov switching models for instance), they assume that the data generating process (DGP) under consideration is driven by an *observed* transition process. This is consistent with our approach of using *common* predetermined observed transition variables that are capable of explaining the dynamics of commodity prices. The use of *common* and *external* transition variables, in addition to the standard autoregressive lags of the dependent variable, is what distinguishes this paper from the majority of the studies in the smooth transition literature.

The paper is organized as follows. Section 2 introduces the STR model, which is technically a regime switching in conditional mean, and applies it to the GYCPI using inflation and oil price as external predetermined transition variables. The section discusses briefly the economic rationale behind selecting inflation and oil price as transition candidates in explaining the nonlinearities in commodity prices through exploring two connections: the commodity price-consumer price connection and the commodity price-oil price connection. Section 3 extends the STR model to characterize the nonlinearity in the conditional variance of the GYCPI. Section 4 applies both the STR in mean and the STR in variance models to the individual price processes forming the GYCPI. Finally, Section 5 concludes.

2 Regime Switching in Mean

2.1 The Smooth Transition Regression Model

The standard STR model of order p is expressed as follows.

$$y_t = \Phi' z_t + \Theta' z_t G(s_t; \gamma, c) + \varepsilon_t, \quad t = 1, 2, \dots, T, \quad (1)$$

where y_t is a scalar, $z_t = (1, y_{t-1}, \dots, y_{t-p})' = (1, \tilde{z}_t)'$, $\Phi' = (\phi_0, \phi_1, \dots, \phi_p) = (\phi_0, \tilde{\Phi}')$, $\Theta' = (\theta_0, \theta_1, \dots, \theta_p) = (\theta_0, \tilde{\Theta}')$, and $\varepsilon_t \sim i.i.d.(0, \sigma^2)$. $G(s_t; \Psi)$ is the transition function. It is a bounded function of the continuous transition variable s_t and it is continuous everywhere in the parameter space for any value of s_t . In practice, the transition variable s_t is an element of the autoregressive lags of the dependent variable, i.e., $s_t = y_{t-d}$ or $s_t = \Delta y_{t-d}$, where $d > 0$ is the delay parameter. In this paper we allow the transition variable s_t to be either an element of the autoregressive lags of y_t or an external variable that is capable of describing the behavior of y_t . The selection of the transition variable, and hence the model type, is performed in the first stage of the modelling process; *the specification stage*. The definition of $G(s_t; \Psi)$ is the one governing the empirical applicability of (1). Some definitions have been suggested in the literature; see, for instance, Granger and Teräsvirta (1993). In this paper, $G(\cdot)$ is a logistic function defined in general as

$$G(s_t; \gamma, \mathbf{c}) = \left(1 + \exp\{-\gamma \prod_{i=1}^k (s_t - c_i)\} \right)^{-1}, \quad \gamma > 0, \quad (2)$$

where γ is the slope of the function, and $\mathbf{c} = (c_1, \dots, c_k)'$ is a vector of location parameters, such that $c_1 \leq \dots \leq c_k$. Given such a definition, the STR model defined in (1) is then referred to as the logistic smooth transition regression (LSTR) model.

The transition function in (2) is a monotonically increasing function of s_t . The restriction $\gamma > 0$ is an identifying restriction. The choice of k is not only crucial in determining the behavior of the logistic transition function, but also has a significant implication in interpreting the time series under consideration. Two common choices for k are used in the literature; $k = 1$ and $k = 2$. In the LSTR model with $k = 1$ (LSTR(1)), the parameters $\Phi + \Theta G(s_t; \gamma, c)$ in equation (1) change monotonically as a function of s_t from Φ to $\Phi + \Theta$. Given such a feature, the LSTR(1) is capable of characterizing asymmetric time series behavior, i.e., processes whose dynamic properties are different in an expansionary regime from what they are in a recessionary regime, and the transition between the two regimes is smooth. The logistic function in the LSTR(1) model takes the form

$$G(s_t; \gamma, c) = (1 + \exp\{-\gamma(s_t - c)\})^{-1}, \quad \gamma > 0. \quad (3)$$

Note that when $s_t \rightarrow -\infty$, $G(\cdot) = 0$; this defines the recessionary regime. On

Figure 2: The smooth transition logistic function of order 1 with a moderate slope $\gamma = 2$ (the solid line) and with an extremely larger slope $\gamma = 1000$ (the dashed line). The threshold value $c = 0.5$.

the other hand, when $s_t \rightarrow +\infty$, $G(\cdot) = 1$ and the time series is said to be in an expansionary regime. The LSTR(1) function in (3) is plotted in Figure 2,

where the threshold $c = 0.5$ and the slope $\gamma = \{2, 1000\}$ for the solid and the dashed lines respectively.

In the LSTR model with $k = 2$ (LSTR(2)), the parameters $\Phi + \Theta G(s_t; \gamma, c)$ change symmetrically around the midpoint $\bar{c} = \frac{c_1 + c_2}{2}$, where the logistic function attains its minimum value. The LSTR(2) model is a three-regime switching regression model in which the dynamics of the two outer regimes, associated with large and small values of s_t , are the same while the behavior in the transition period (middle regime) is different. The second-order logistic function in the LSTR(2) model takes the form

$$G(s_t; \gamma, c) = (1 + \exp\{-\gamma(s_t - c_1)(s_t - c_2)/\sigma^2\})^{-1}, \quad \gamma > 0, \quad c_1 \leq c_2, \quad (4)$$

where $\gamma > 0$ and $c_1 \leq c_2$ are identifying restrictions as mentioned before. The function achieves its minimum value $G_{\min} = (1 + \exp\{-\gamma\tilde{c}\}/\sigma^2)^{-1}$, where $\tilde{c} = c_1 c_2 - \bar{c}^2$ and $\bar{c} = \frac{c_1 + c_2}{2}$, when the transition variable s_t is equal to $\bar{c}$. The LSTR(2) function in (4) is plotted in Figure 3, where $c_1 = -1, c_2 = 2$ and $\gamma = \{1, 1000\}$ for the solid and dashed lines respectively. Observe how the functions change symmetrically around $\bar{c} = 1/2$ (the midpoint). When $\gamma = 0$, the transition function $G(s_t; \gamma, c_1, c_2) = 1/2$, and the STR model in (1) nests the linear model. When $\gamma \rightarrow \infty$ in the LSTR(1) model, the result approaches the pure threshold model; when $\gamma \rightarrow \infty$ in the LSTR(2) model, the result is a pure threshold model with three regimes such that the two outer regimes are similar while the ground regime (middle regime) is different and the transition between regimes is swift (see the dashed line in Figure 3).

I shall follow the modelling framework proposed by Granger and Teräsvirta (1993), Teräsvirta (1994), and Eitrheim and Teräsvirta (1996), which consists of three stages. The first stage is the *specification stage*, in which a linear autoregressive model, $AR(p)$, where p is the value that minimizes the Akaike (1974)

Figure 3: The smooth transition logistic function of order 2 with a moderate slope $\gamma = 1$ (the solid line) and with an extremely larger slope $\gamma = 1000$ (the dashed line). The threshold values are $c_1 = -1$ and $c_2 = 2$.

information criterion (AIC) directly, is specified. Before accepting the suggested $AR(p)$ model as the starting point of the STR analysis, diagnostic tests should be applied to the model to ensure its adequacy as a starting model. In particular, the Ljung-Box (1978) test of no serial correlation of order $q = 1$ up to $q = 8$ in the residuals, $Q(q)$, and Engle's (1982) Lagrange multiplier (LM) test of no autoregressive conditional heteroskedasticity of order $v = 1$ up to $v = 4$ in the residuals, $LM_{ARCH(v)}$, are considered. If the model passes those tests, we move to the next phase in the specification stage which is *nonlinearity testing*. The problem with nonlinearity testing is that the nonlinear STR model considered is only identified under the alternative hypothesis. Luukkonen, Saikkonen, and Teräsvirta (1988), based on a paper by Davies (1977), suggested a solution to this problem. Their solution, which was adopted by Teräsvirta (1994), was simply to replace the transition function (2) in (1) by a Taylor approximation about the null hypothesis $\gamma = 0$. In particular they assumed a first order logistic function, i.e., $k = 1$ in (2), and performed a third order Taylor approximation

about the null hypothesis $\gamma = 0$. The approximation yields

$$y_t = \Phi' z_t + \frac{1}{4} \gamma \Theta' z_t (s_t - c) - \frac{1}{48} \gamma^3 \Theta' z_t (s_t - c)^3 + \varepsilon_t. \quad (5)$$

Using $z_t' = (1, \tilde{z}_t')$, $\Phi' = (\phi_0, \tilde{\Phi}')$, and $\Theta' = (\theta_0, \tilde{\Theta}')$, and reparameterizing, equation (5) can be expressed as

$$y_t = \delta_0 + \delta_1' \tilde{z}_t + \pi_1' \tilde{z}_t s_t + \pi_2' \tilde{z}_t s_t^2 + \pi_3' \tilde{z}_t s_t^3 + \varepsilon_t^*, \quad (6)$$

where $\varepsilon_t^* = \varepsilon_t + R(\gamma, c, s_t)$; $R(\cdot)$ being the remainder and π_j , $j = 1, 2, 3$, is of the form $\gamma \tilde{\pi}_j$, where $\tilde{\pi}_j \neq 0$ is a function of $\tilde{\Theta}$. The null hypothesis of linearity is then $H_{0L} : \pi_1 = \pi_2 = \pi_3 = 0$. Also note that because $\varepsilon_t^* = \varepsilon_t$ under the null hypothesis, the asymptotic theory will not be affected if an LM test is used. Following Luukkonen et al. (1988) and Teräsvirta (1994), a convenient procedure for computing the LM statistic by OLS is to estimate (6) under the null hypothesis and compute the sum of squares of the residuals (SSR_0), then estimate (6) under the alternative hypothesis and compute SSR_1 . The LM statistic is computed as $LM = \frac{T(SSR_0 - SSR_1)}{SSR_1}$. The test statistic has an asymptotic χ^2 -distribution with $3p$ degrees of freedom when the null hypothesis is valid. However, the F -statistic is recommended because the χ^2 -statistic can be size-distorted in small and even moderate samples. In this paper, we shall use the F -distribution with $3p$ and $T - 4p - 1$ when the null hypothesis H_{0L} is valid. The test is repeated for each transition candidate in the transition set. If the null hypothesis of linearity, H_{0L} , using the F -test (F_L) is rejected for at least one of the models, the model against which the rejection, measured in the p -value, is strongest is chosen to be the STR model to be estimated.

Another purpose of conducting the linearity test is to use the test results for model selection. If linearity is rejected and a transition variable is selected,

the next step is to choose a model type, i.e., to choose between LSTR(1) or LSTR(2) models. The choice between the two models can be based, again, on the auxiliary regression (6). Teräsvirta (1994) showed that when $c = 0$ then $\pi_2 = 0$ when the model is an LSTR(1), where as $\pi_1 = \pi_3 = 0$ when the model is an LSTR(2). The following F -tests sequence was then suggested based on the auxiliary regression in (6):

1. Test the null hypothesis: $H_{04} : \pi_3 = 0$ with an ordinary F -test (F_4). A rejection of H_{04} can be interpreted as a rejection of the LSTR(2).
2. Test the null hypothesis that $\pi_2 = 0$ given that $\pi_3 = 0$, $H_{03} : \pi_2 = 0 | \pi_3 = 0$, using another F -test (F_3). Failure to reject H_{03} indicates that the model is an LSTR(1).
3. The last F -test (F_2) in the sequence is to test the null hypothesis that $\pi_1 = 0$ given that $\pi_2 = \pi_3 = 0$ as, $H_{02} : \pi_1 = 0 | \pi_2 = \pi_3 = 0$. Rejecting H_{02} after accepting H_{03} supports the choice of the LSTR(1) model. Accepting H_{02} after rejecting H_{03} points to the LSTR(2) model.

After carrying out the three F -tests and noting which hypotheses are rejected, if the test H_{03} yields the strongest rejection measured in the p -value, choose the LSTR(2) model; otherwise select the LSTR(1) model.

After specifying the type of the STR model and determining the delay parameter, the next step is to estimate the model. Estimation is carried out using maximum likelihood method conditional on the $\Psi = \{\gamma, c\}$ parameters. The conditional log-likelihood is maximized using the iterative Broyden-Fletcher-Goldfarb-shanno (BFGS) algorithm. Starting-values for γ and c needed for the algorithm are obtained by constructing a three dimensional grid.³ The model

³To facilitate the construction of an effective grid, I follow Teräsvirta's (1998) suggestion to standardize the exponent of the transition function $G(s_t; \gamma, c)$ by dividing it by the k^{th} power of the sample standard deviation of the transition variable $\hat{\sigma}_s^k$. This is done mainly to render the parameter γ scale-free.

parameters are estimated conditionally on γ and c , and the sum of squared residuals is computed. The procedure is repeated for N combinations of these parameters. The parameter values that minimize the sum of square residuals are selected.

The last stage in the modelling procedure is the *diagnostic stage* where the adequacy of the fitted model is considered. The mis-specification tests for the STR models that have been considered in Eitrheim and Teräsvirta (1996) and Teräsvirta (1998) will be considered in this paper.

2.2 The Threshold Variable in the STR Model

The selection of the transition variable s_t in the STR models is crucial as it explains the dynamics of the dependent variable y_t , which is the ultimate goal of fitting such a model. The majority of the researchers in the regime switching literature start their analysis by suspecting a nonlinear pattern in the DGP and conducting a sequence of linearity tests to confirm it. Once nonlinearity is proven to be present, they are faced with the decision of which transition variable they should employ. Most of the studies employ a lag order autoregressive component of the dependent variable as transition variable. This is probably because it is difficult to find external transition variables that can explain the dynamics of the DGP, especially when the economic theory does not say much about the relation between the dependent variable under consideration and the potential transition candidates. In this paper, in addition to the autoregressive lags of the dependent variable, other potential transition candidates that can explain the detected nonlinearities in commodity prices are considered. Since commodity markets respond to business cycle fluctuations, it was natural to consider, among the external transition candidates, business cycle variables. In particular, current and one period lag unemployment, money supply measures

(M1 and M2), current and one period lag interest rates (1, 5, and 10 years U.S. treasury bills), and oil price were considered. Nonlinearity was rejected for all variables except for oil price and the U.S. inflation rates and, therefore, both variables were selected as the potential transition candidates. The rationale behind using inflation rates and the price of oil lies in their connection with commodity prices. This connection is briefly discussed in the following subsection.

2.3 Commodity Price-Consumer Price and Commodity Price-Oil Price Connections

The commodity price-consumer price connection rests on a number of linkages. The most important one describes commodity prices as assets whose price react quickly to unanticipated shocks. For instance, a surge in aggregate demand in response of a change in money supply causes commodity prices to initially overshoot their long run equilibrium while final goods prices only respond with a lag (Blomberg & Haris, 1995). This overshooting hypothesis has been used by many authors (see Frankel (1986), Boughton and Branson (1991), and Fuhrer and Moore (1992)) to explain the commodity price-consumer price connection. The previous connection implies one way causality from commodity prices to consumer prices. However, a feedback relation also exists. In a recent paper, Kyrtsov and Labys (2006), using monthly U.S. data, found evidence of *nonlinear bi-directional* (feedback) Granger causality between the growth rate (logarithmic differences) of the consumer price index and the commodity price index. We make use of this bi-directional Granger causality to justify the use of inflation rate as a potential transition variable that is capable of explaining the dynamics of commodity prices.

The connection between the price of a commodity and the oil price rests on

the term of selling used in the contract effected between the buyer and the seller of the commodity. The frequently used terms of selling are *free on board* (FOB) prices and *cost insurance & freight* (CIF) prices. In addition to the insurance cost, the CIF price of a commodity includes the transportation cost (ocean freight). Since a significant portion of the ocean freight is due to the bunker fuel cost, one would expect oil price to play a significant role in explaining the behavior of commodity prices recorded on a CIF basis. The fuel cost represents roughly around 20 to 25% of the recorded CIF price of a commodity (Fahmy, 2011). Seven out of the 24 commodities forming the commodity price index studied in this paper were recorded on a CIF basis and, therefore, oil price is considered a potential transition candidate for modelling nonlinearities in these commodities as well as the commodity price index. It turned out, as we shall see in Section 4, that oil price is indeed the best transition variable that was capable of explaining the observed nonlinear dynamics in all commodity prices recorded on a CIF basis.

2.4 The Data

The commodity price index studied in this paper is an index of 24 primary commodity prices developed by Grilli and Yang (1988) and known as the GYCPI. In this paper, we use the extended version of the Grilli and Yang series from 1900 to 2007, developed and provided by Pfaffenzeller, Newbold, and Rayner (2007).⁴ we follow the convention in the literature and use the logarithm of the real index defined as $y_t = \log \left(\frac{GYCPI_t}{MU_t} \right)$. A plot of y_t is shown in the top panel of Figure 4. The first potential transition candidate is inflation rate defined as the first difference of the logarithm of the U.S. consumer price index for all urban consumers (CPI_t), $\Pi_t = \Delta \log(CPI_t)$. The second transition candidate

⁴I am indebted to Stephan Pfaffenzeller for supplying the recent update from 2003 to 2007.

Figure 4: The logarithm of real GY CPI, $y_t = \log(\frac{GYCPI_t}{MUV_t})$, U.S. inflation rate, $\Pi_t = \log(CPI_t)$, and the logarithm of real U.S. crude oil price, $R_t = \log(\frac{Oil\ price_t}{CPI_t})$, between 1900 and 2007.

is the logarithm of real crude oil price defined as $R_t = \log\left(\frac{Oil\ price_t}{CPI_t}\right)$. The stationarity of y_t is confirmed at the 5% level of significance by the ADF, PP, and KPSS tests (see Table 1).

A plot of y_t against our two transition candidates between 1900 and 2007 is shown in Figure 4. By looking closely at the figure, one can immediately observe the distinction between the behavior of inflation rate and oil price in three periods: (1) In the *early period*, between 1900 and 1950, the fluctuations in inflation rate are higher in magnitude compared to those in oil prices; (2) in the mid period, 1950 to 1970, there is not much to tell as both variables look steady with very small fluctuations; (3) in the *late period*, 1970 to 2007, however, the oil price time series exhibits higher swings, especially around the two well known oil shock periods (1974 and 1984), as compared to the inflation rate, which

seems to have stabilized after the second oil shock of 1984. This simple analysis suggests a reverse pattern between both variables. That is, if a connection between the GYCPI and both variables is established, one, by using inflation rate alone as the transition variable, will adequately capture the early period fluctuations in commodity prices, while, using oil price, late period variations will be well modelled. In the following subsections, Teräsvirta's (1994) three stages modelling procedure for the LSTR models will be applied to the GYCPI using inflation and oil price as transition variables respectively.

2.5 STR in Mean Model Applied to the GYCPI

2.5.1 Switching Variable: Inflation

The AIC for the logarithm of real GYCPI is minimized at a lag order $p = 1$ and, therefore, the starting linear model for the nonlinearity analysis is an AR(1). The model passes all mis-specification tests in the preliminary stage and, therefore, acts as a good starting point to our nonlinearity analysis. Moving to the next step, which is nonlinearity testing, the set of the predetermined potential transition variables, Ω , in this case consists of the autoregressive variable y_{t-1} and its first difference, Δy_{t-1} , and the current and the one period lag inflation (Π_t and Π_{t-1} respectively) as the potential transition candidates, i.e., $\Omega = \{y_{t-1}, \Delta y_{t-1}, \Pi_t, \Pi_{t-1}\}$ for $t = 1, \dots, T$. The sequence of nonlinearity tests mentioned in Section 2 was executed for each of the potential transition variables in Ω and the results are reported in Table 2. At a first glance, the current inflation rate, Π_t , tagged with the symbol **, seems to be the variable with the strongest test rejection (the smallest p -value; see the second column of Table 2) and, therefore, it should be selected as the appropriate transition variable for the STR model. The associated model is clearly the LSTR(2) model as the p -value of the F_3 test is less than the p -value of the F_2 and F_4 . However, using

$s_t = \Pi_t$, and proceeding to the estimation stage, the diagnostics of the estimated LSTR(2) model showed, besides some residuals autocorrelations of order 2 and 3, severe parameter change, i.e., the parameter constancy hypothesis was rejected, which is a clear indicator of mis-specification. This is, most probably, due to the possible endogeneity resulting from the correlation between the period t commodity price index and the period t consumer price index, which might renders the estimates unreliable. Therefore, a better choice is to use the one period lag inflation instead of current inflation as transition variable. Actually, applying our testing procedure confirms that; the second best transition candidate is the one period lag inflation Π_{t-1} , tagged with the symbol * in Table 2, and the suggested model is the LSTR(1). The p -values of the F -tests for linearity and model selection sequence previously described are reported in the same table.

The best fitted LSTR(1) model that I managed to obtain after dropping the insignificant drift parameter in the nonlinear part of y_t and the results of Eitrheim and Teräsvirta (1996) and Teräsvirta's (1988) mis-specification tests are reported as follows.

$$y_t = 0.009 + 0.98 y_{t-1} - \{0.44 y_{t-1}\} (1 + \exp(-\frac{213}{1278} (\Pi_{t-1} - \frac{0.07}{0.003}) / 0.046))^{-1} + \hat{\varepsilon}_t, \quad (7)$$

$$\overline{R}^2 = 0.88, \quad \hat{\sigma}_{\Pi} = 0.046, \quad \hat{\sigma} = 0.10,$$

$$AUTO(1) = 0.29, \quad AUTO(8) = 0.82, \quad LM_{ARCH(1)} = 0.26,$$

$$LM_{ARCH(4)} = 0.87, \quad PC(1) = 0.19, \quad PC(2) = 0.43, \quad PC(3) = 0.15,$$

$$NRNL = 0.21, \quad JB = 0.001, \quad K_3 = -0.5, \quad K_4 = 4.13,$$

where $\hat{\sigma}_{\Pi}$ is the sample standard deviation of the transition variable $s_t = \Pi_{t-1}$ and $\hat{\sigma}$ is the residual standard deviation. The figures in parentheses beneath

the parameter values are standard deviations. $AUTO(v)$ is the p -value for the v th order autocorrelation. $LM_{ARCH(q)}$ is the p -value of the q th order $ARCH$. $PC1$, $PC2$, and $PC3$ are p -values for parameter constancy tests against monotonic change, non-monotonic symmetrical change, and non-monotonic and non-symmetrical change respectively. $NRNL$ is the p -value for the no remaining nonlinearity test. JB is the p -value of the Jarque-Bera's (1987) test of normality. K_3 is skewness and K_4 is kurtosis. Perhaps the most noticeable detail of (7) is the large standard deviation of the estimated slope of the logistic function, $\hat{\gamma} = 213$. It is common, for LSTR models, that the estimated standard deviation of γ tends to be large for large values of γ . This is not crucial, however, as it will not affect either the shape of the logistic function $G(\cdot)$ or the other estimates of the model. The message delivered by this enormous slope of the transition function is that y_t will be moving sharply from one regime to the other. This is not surprising since we are modelling the dynamic behavior of a commodity price index, which is flexible by nature. This type of behavior can be observed from the transition function $G(\Pi_{t-1}, \gamma, c)$ plotted in Figure 5. Other than the large slope of the transition function, the estimated coefficients are all significant and the model passes the misspecifications tests mentioned before. One exception is the rejection of the null-hypothesis of normality of errors at the 5% level of significance as seen from the p -value of the JB test statistic. But, this is due to the existence of outliers in the time series. A quick look at the standardized residuals time series plotted in Figure 6, one can notice the very large outliers in 1921 and 1937 where the absolute value of the standardized residuals is greater than three. These outliers are corresponding to the post WWI and the post great depression periods respectively. This is consistent with the findings of Cuddington and Urzua (1989) and Powell (1991) who both fitted a DS model to y_t accounting for structural break in 1921 and 1937 respectively.

The threshold value $\hat{c} = 0.07$ indicates that if the one period lag inflation rate increases beyond 7%, the logarithm of the real commodity price index will move to the *upper* stationary regime with unconditional mean of 0.02 and unconditional variance of 0.014. On the other hand, if there is a slow down in economic activities so that inflation decreases below the 7% threshold, the real commodity index will switch to the lower regime with 0.5 unconditional mean and a larger unconditional variance equals to 0.25. The higher variance of the lower regime (17 times larger as compared to the upper regime) indicates that the logarithm of the real commodity price index likes to wander in the recessionary regimes but always revert back to the mean because the time series is stationary. This dynamic behavior can be detected by examining the transition function and the behavior of the transition variable above and below the estimated threshold value, i.e., when $s_t = \Pi_{t-1} = 7\%$, as shown in Figure 7. The behavior of the transition function captures the dynamics of the transition variable Π_{t-1} during the periods posts the WWI and WWII. It also exhibits the high swings in inflation during the two well known oil shocks in 1974 and 1984. No fluctuations are present during the great depression of the 1930's, however. This implies that the inflation rate is capable of capturing the fluctuations in commodity prices mainly during the early period (between 1900 and 1950); observe that the GYCPI, starting from the early fifties till 2007, is following one contractionary regime (with the exception of the two oil shocks). Therefore, inflation by itself fails to account for the observed late period fluctuations in the GYCPI when nonlinearity is modelled in the conditional mean. This is, perhaps, due to the continuous monitoring of the FED to the U.S. inflation rate as part of its monetary policy.

Figure 5: Transition function $G(\Pi_{t-1}, \gamma, c)$ as a function of observations. Each dot corresponds to an observation. The transition variable is the one period lag inflation.

Figure 6: Standardized residuals of the fitted LSTR1 model between 1900 and 2007 with one period lag inflation rate as transition variable.

Figure 7: A plot of the transition variable Π_{t-1} and the transition function $G_t(\Pi_{t-1}; \gamma, c)$ between 1900 and 2007.

2.5.2 Switching Variable: Oil Price

We have just seen how inflation captured the early fluctuations observed in the GYCPI. In this section, the analysis is repeated using the logarithm of real crude oil, R_t , as the predetermined transition variable. The set of predetermined transition variables in this case is $\Xi = \{y_{t-1}, \Delta y_{t-1}, R_t, R_{t-1}\}$ for $t = 1, \dots, T$. The nonlinearity tests sequence in the previous section was executed for each variable in Ξ . The p -values of the F -tests for linearity and model selection are reported in Table 3. The best transition candidate in Ξ , tagged with the symbol * in Table 3, is the one period lag of the logarithm of real oil price, R_{t-1} , and the suggested model is the LSTR(1).

The best fitted LSTR(1) model that I managed to obtain for y_t and the mis-specification tests results are reported as follows.

$$y_t = \underset{(0.01)}{0.009} + \underset{(0.04)}{0.98} y_{t-1} - \left\{ \underset{(0.04)}{0.10} + \underset{(0.14)}{0.40} y_{t-1} \right\} (1 + \exp(-\underset{(5.23)}{7.3} (R_{t-1} + \underset{(0.06)}{3.1}) / 0.38))^{-1} + \hat{\varepsilon}_t, \quad (8)$$

$$\bar{R}^2 = 0.88, \quad \hat{\sigma}_R = 0.38, \quad \hat{\sigma} = 0.10,$$

$$AUTO(1) = 0.12, \quad AUTO(8) = 0.71, \quad LM_{ARCH(1)} = 0.91,$$

$$LM_{ARCH(4)} = 0.78, \quad PC(1) = 0.13, \quad PC(2) = 0.32, \quad PC(3) = 0.51,$$

$$NRNL = 0.15, \quad JB = 0.26, \quad SK = -0.29, \quad K = 3.5.$$

Unlike the pervious case, the moderate slope of the transition function, $\hat{\gamma} = 7.3$, implies a smooth transition between the two regimes (see Figure 8). The estimated threshold value $\hat{c} = -3.1$ means that if the one period lag logarithm of real oil price is higher than -3.1 , or, in other words, if the value of the one period lag oil price exceeds 5% of the CPI, the real commodity index will move to the *upper* stationary regime with unconditional mean of -0.2 and

unconditional variance of 0.015. On the other hand, if the percentage of oil price out of CPI is less than 5%, the real commodity index will behave like a near random walk with a positive drift, with 0.5 unconditional mean and a larger unconditional variance equals to 0.25. The larger variance of the lower regime (17 times larger as compared to the upper regime) indicates that the logarithm of the real commodity price index likes to wander in the recessionary regimes but always revert back to the mean because the time series is stationary. This is exactly the same behavior that y_t has exhibited when the one period lag inflation was used as a transition variable. This result motivates the use of external transition variables in regime switching models and confirms the claim that both variables (oil and inflation) are capable of explaining the nonlinearity in the GYCPI. Inflation captured the early fluctuations in the Grilli and Yang index, but failed to capture the late ones; oil price, on the other hand, captured the late dynamics, but failed to capture the early ones. Therefore, both variables can be seen as *complements* in explaining the dynamic behavior of the GYCPI. This complementarity between oil and inflation can be seen from their transition functions in Figure 9. During the early period (between 1920s and the early 1970s), the price of oil displayed one regime (lower regime) and, thus, failed to capture the early dynamics of the commodity index, whereas, inflation rate switched from one regime to the other during this period and, therefore, succeeded in characterizing the early nonlinear dynamics in the price index. In the later period, the situation was reversed; inflation displayed one lower regime, whereas, oil price switched between the upper and the lower regimes capturing the recent nonlinearities in the commodity price index.

Figure 8: Transition function $G(R_{t-1}, \gamma, c)$ as a function of observations. Each dot corresponds to an observation. The transition variable is the one period lag logarithm of real crude oil.

Figure 9: Panel (b) and (c) show the first order logisitic transition function with the one period lag inflation rate and the one period lag logarithm of real oil price as transition variables respectively.

3 Regime Switching in Variance

In this section, we extend the regime switching framework discussed in Section 2 to model the regime switching in the conditional variance of the GYCPI. The STR in variance was adopted by some authors; see, for instance, Lee and Degennaro (2000), Lundbergh and Teräsvirta (1998), and Lubrano (2001). We follow a somewhat similar approach in modeling the regime switching in variance. we use the smooth transition logistic function in modelling the dynamics of the regimes but we do not restrict the transition variable to be one of the lagged innovations or squared innovations as in the previously mentioned studies; we also consider the possibility that the transition variable could be an external variable such as inflation and oil price.

3.1 The Smooth Transition Regression in Variance Model

Let $y_t = \Phi' w_t + \xi_t$, where y_t is a scalar, $w_t = (1, y_{t-1}, \dots, y_{t-p})'$, $\Phi' = (\phi_0, \phi_1, \dots, \phi_p)$, and let $\xi_t = Z_t \sqrt{h_t}$, with $Z_t \sim i.i.d.(0, 1)$. Define the conditional variance equation as

$$h_t = \alpha_1 F(s_t; \gamma, c) + \alpha_2 (1 - F(s_t; \gamma, c)), \quad (9)$$

where α_1 and α_2 are parameters and $F(s_t; \gamma, c)$ is a first or second order logistic transition function defined as in (3) and (4) respectively. The transition variable could be one of the past innovations ξ_{t-i} , or the squared innovations ξ_{t-i}^2 , or even an external variable.

The STR in variance model allows for time-varying shifts in the parameters of the conditional variance equation. The model also allows for asymmetric response of the conditional variance equation to positive versus negative news. The news in this context represent the change in the behavior of the transition variable. Suppose, for instance, that y_t is a commodity price index and the ex-

ternal transition variable is the one period lag inflation rate ($s_t = \Pi_{t-1}$). In case of a negative news, i.e., in case that $s_t > c$, the transition function approaches 1 and the conditional variance $h_t = \alpha_1$, whereas in case of a positive news, $h_t = \alpha_2$. The two parameters α_2 and α_1 in equation (9) change monotonically as a function of s_t from α_2 to α_1 and allows for asymmetric effect of external news on the conditional variance of the commodity price index. To ensure the non-negativity of the conditional variance equation, we impose the restrictions that $\alpha_1 > 0$ and $\alpha_2 > 0$.

Following the nonlinearity analysis framework developed in Section 2, we test the null hypothesis that the conditional variance of the commodity price index is linear against the alternative that it follows the STR model defined in (9). Using the one period lag inflation, $\Pi_{t-1} = \Delta \log(CPI_{t-1})$, and the one period lag of the logarithm of real oil price, $R_{t-1} = \log\left(\frac{Oil\ price_{t-1}}{CPI_{t-1}}\right)$, as our transition candidates, the STR in variance model is applied and estimated in each case in the following subsections.

3.2 STR in Variance Model Applied to the GYCPI

3.2.1 Switching Variable: Inflation

Consider the specification in (9) with $s_t = \Pi_{t-1} = \Delta \log(CPI_{t-1})$. Applying the nonlinearity test discussed in the previous section, the computed $SSR_0 = 0.06849$ and the $SSR_1 = 0.061224$. The computed F -statistics equals 4.11 whereas, at the 5% level of significance with 3 and 104 degrees of freedom, the tabulated $F = 2.70$. Therefore, we reject the null hypothesis that the conditional variance equation is linear against the alternative of an STR in variance model with lag inflation as transition variable. The estimated model

and the mis-specification tests results are reported as follows.

$$y_t = \underset{(0.01)}{0.001} + \underset{(0.03)}{0.97} y_{t-1} + \hat{\xi}_t; \quad h_t = \underset{(0.004)}{0.02} F_t + \underset{(3 \times 10^{-4})}{0.006} (1 - F_t), \quad (10)$$

$$F_t(\Pi_{t-1}, \gamma, c) = \left(1 + \exp\left(-\underset{(19.6)}{82} (\Pi_{t-1} - \underset{(6 \times 10^{-4})}{0.03}) / 0.047 \right) \right)^{-1},$$

$$Q(1) = 0.18, \quad Q(8) = 0.55, \quad K_3 = -0.4, \quad K_4 = 3.9, \quad JB = 0.06,$$

where $\hat{\sigma}_\Pi = 0.047$ is the standard deviation of the transition variable. All model parameters are significant as seen from the standard deviations reported. The estimated slope of the transition function is large, $\hat{\gamma} = 82$ indicating that the conditional variance moves swiftly from one regime to another. This can be viewed from the transition function plotted in Figure 10. The estimated threshold parameter of the STR in variance model $\hat{c} = 3\%$ is slightly lower than the estimated threshold in the STR in mean model (7%). Both the transition variable and the transition functions of the STR in mean model and the STR in variance model are plotted in Figure 11. The two estimated threshold parameters from both models are also shown in Panel (a) in Figure 11. A closer look at the transition function, $F(\cdot)$ in Figure 11, Panel (b), one can observe roughly the same dynamics observed in the transition function, $G(\cdot)$, of the regime switching in mean model developed in Section 2 (see Panel (c), Figure 11). Observe how both transition functions display the same dynamics during the periods 1918 till 1921, during the 1940s and early 1950s, and during the two oil shocks in 1974 and pre 1984. Also in the late period, both functions exhibit one lower regime. We can conclude that inflation as a transition variable in both models (the STR in mean and the STR in variance) failed to capture the *late dynamics* in the commodity price index time series. This result implies that the two models can be seen as *substitutes* when modelling nonlinearity in

the commodity price index using inflation as threshold variable. Although the transition functions of both models displayed similar dynamics in the limiting regimes, it should be noted that the two models (the STR in mean and the STR in variance) explain the behavior of commodity prices from two different angles: in the regime switching in variance model, the transition function, $F(\cdot)$, is modelling switching regimes in the conditional variance equation while the mean equation is unchanged, whereas, in the regime switching in mean model, the transition function, $G(\cdot)$, is modelling switching regimes in mean leaving the variance equation unchanged. Therefore, the former explains how the variance of the GYCPI responds to fluctuations in lag inflation rate around a threshold, while the later explains how the index itself does so. Table 4 summarizes the results obtained from fitting both models.

Figure 10: The transition function $F(\Pi_{t-1}, \gamma, c)$ as a function of observations. Each dot corresponds to an observation. The transition variable is the one period lag inflation.

3.2.2 Switching Variable: Oil

In this subsection, we re-estimate the STR in variance model using the one period lag logarithm of real oil price as the transition variable. The estimated

Figure 11: A plot of the transition variable Π_{t-1} (panel (a)), the transition function from the STR in variance model (panel (b)), and the transition function from the STR in mean model (panel (c)).

model and the mis-specification tests results are reported as follows.

$$y_t = \underset{(0.01)}{0.005} + \underset{(0.03)}{0.95} y_{t-1} + \hat{\xi}_t; \quad h_t = \underset{(0.01)}{0.03} F_t + \underset{(0.01)}{0.008} (1 - F_t), \quad (11)$$

$$F_t(R_{t-1}, \gamma, c) = \left(1 + \exp\left(-\underset{(23.8)}{17} (R_{t-1} + \underset{(0.07)}{3.1})/0.38\right) \right)^{-1},$$

$$Q(1) = 0.27, \quad Q(8) = 0.35, \quad K_3 = -0.3, \quad K_4 = 3.9, \quad JB = 0.05,$$

where $\hat{\sigma}_R = 0.38$ is the standard deviation of the transition variable. The moderate slope of the transition function $\hat{\gamma} = 17$ suggests a smooth transition between the regimes. The transition function is plotted in Figure 12.

In case of a negative news (upper regime), i.e., when the oil price is above its threshold value, the transition function $F_t(R_{t-1}; \gamma, c)$ tends to 1 and the conditional variance of the error term $h_t = 0.03$, which is approximately the same result that was obtained in the STR in variance model with inflation as transition variable (see Table 5). On the other hand, in case of a positive news (lower regime), the oil price is below its threshold value and the conditional variance $h_t = 0.008$, which is, again, approximately equal to the same value obtained from the STR in variance model with inflation as transition variable. This result implies that the price of oil as a transition variable in both the STR in mean and the STR in variance models failed to capture the *early dynamics* in the commodity price index. Figure 13, Panel (b), shows the transition function in the STR in variance model with oil as transition variable. Observe how the function captures only the late dynamics after the 1974 oil shock. This same conclusion was reached when the price of oil was used in the STR in mean model (see Figure 13, Panel (c)). Table 5 summarizes the results obtained from fitting both models with $s_t = R_{t-1}$. These results also imply that the two models can be seen as *substitutes* when modelling nonlinearity in the commodity price index

Figure 12: The transition function $F(R_{t-1}, \gamma, c)$ as a function of observations. Each dot corresponds to an observation. The transition variable is the one period lag logarithm of real oil price.

using the oil price as threshold variable.

To sum up, we can conclude that both models (the STR in mean and the STR in variance) can be seen as *substitutes* when modelling nonlinearity in the GYCPI and the transition variables (the inflation rate and the price of oil) can be seen as *complements* in characterizing its nonlinear dynamics in the sense that inflation is capable of capturing the early dynamics whereas the price oil can capture the late ones. The reason we don't have only one common variable that can explain the behavior of the GYCPI is that the index itself consists of 24 individual price series each of which was recorded based on a different border price (FOB, CIF, settlement, and auction price). Further insight can be gained by classifying the 24 individual price series into groups according to their border prices and then modelling nonlinearity in each group using a common transition variable. This is the subject matter of the following section.

4 Regime Switching in Individual Commodities

We have just seen how the price of oil and the inflation rate were capable of modelling the nonlinearities in the GYCPI. The question now is which tran-

Figure 13: A plot of the transition variable R_{t-1} (panel (a)), the transition function from the STR in variance model (panel (b)), and the transition function from the STR in mean model (panel (c)).

sition variable should we employ when modelling nonlinearities in *individual* commodities? This issue is directly linked to the observed phenomenon that commodity prices tend to move together in groups in response to a common macroeconomic variable or group of variables. We attempt to explain this co-movement by, first, classifying commodity prices into groups according to their border price, and then trying to find the best common transition variable that can explain the dynamic behavior of each group.

The rationale behind the border price classification of commodity prices rests on how commodity prices are recorded and it can provide an insight regarding the selection of the transition variable that is capable of modelling their dynamics. For the traded commodities, the standard practice followed by the major institutions when recording data on commodity prices is to select the largest trading route of a commodity and detect whether the trade volume is controlled by a major exporting or importing country. If this route is dominated by a major exporter, the export price at the exit point of the exporting country (FOB price) will be recorded; if the route, on the other hand, is dominated by a major importer of the commodity, the import price at the entry point of the importing country (CIF price) will then be recorded. Of course there are some exceptions, but this is usually the standard practice. For the other commodities that are traded on exchanges (metals and grains mostly), where physical delivery of the commodity is not a must, settlement prices, option prices, or auction prices are used.

This way of recording commodity data suggests that, for commodities that are recorded on a FOB basis, the transition variable is more likely to be one of the macroeconomic news variables, inflation for instance, in the *exporting* country. Notice here that the price of oil can not be considered among the potential transition candidates in this group of commodities. The reason is that

FOB prices do not include the ocean freight which is driven by the price of oil. On the other hand, for commodities that are recorded on a cost and freight or CIF basis, the suggested transition variable is either the price of oil or one of the macroeconomic news variables in the *importing* country. The reason behind choosing oil price for this CIF group of commodities is that a significant portion of it is due to freight cost (shipping cost), which, in turn, is driven by the price of crude oil. Therefore, a reasonable hypothesis here is that ocean freight and, hence, oil price plays a significant role in modelling the dynamics of those commodities falling in this group.

Our results confirm the previous rationale of selecting the transition variable for individual commodities based on their border price classification. Inflation rate and the price of oil were the transition variables that showed the highest rejection of linearity tests for all FOB and CIF commodities respectively (see Table 6). Details regarding the classification, estimation, and the dynamic analysis of the limiting processes of the 24 individual commodity series forming the GYCPI are found in Fahmy (2011).

5 Concluding Remarks

In this paper we followed the empirical approach in modelling the dynamics of commodity prices. The approach was motivated by the observation that commodity prices tend to move together in groups in response to a common macroeconomic variable or group of variables. We attempted to explain this phenomenon by classifying commodity prices according to their border price (an issue that has been ignored in previous studies), and then by attempting to find the best common macroeconomic variable that is responsible for this co-movement. The rationale behind the border price classification rests on how the border price can provide an insight regarding the macroeconomic variable that

drives the individual price series in each group. The price of oil was the best transition variable that was capable of modelling the nonlinearity in the price processes that were recorded on a cost and freight basis. The rationale behind using oil price in this group rests on the commodity price-oil price connection, which suggests that since a significant portion of the cost and freight price is due to freight cost (shipping cost), which, in turn, is driven by the price of crude oil, therefore, oil price plays a significant role in modelling the dynamics of those commodities falling in this group. Our results confirmed this connection.

As opposed to most of the previous studies that used the past autoregressive lags of the dependent variable as transition variables in modelling the nonlinearities in price processes, we used external potential transition variables that were capable of explaining such nonlinear dynamics. In particular, two transition variables were proven successful in explaining the behavior of commodity prices: inflation rate and oil price.

The study attempted to model the nonlinearity in both the conditional mean and the conditional variance of the price series. Hence, the STR in mean and the STR in variance models. Both models were fitted to the GYCPI using the two potential transition candidates (inflation and oil price). The analysis was also extended to model the nonlinearities in the individual price series forming the index.

In case of the commodity index, using either inflation rate or oil price as potential transition candidate, both models (the STR in mean and the STR in variance) displayed the same dynamics in the limiting processes of the commodity index. When nonlinearity was modelled in the conditional mean (STR in mean model), leaving the conditional variance equation constant, the conditional mean equation of the commodity index displayed a stationary AR(1) behavior in the upper regime and a near random walk with a drift parameter

in the lower regime. The behavior did not change with either inflation or oil price. When nonlinearity was modelled in the conditional variance equation of the commodity index (STR in variance model), leaving the conditional mean equation unchanged, the conditional variance was approximately three times higher in the upper regime as opposed to the lower regime. These results suggest that both models can be seen as *substitutes* when modelling nonlinearity in the commodity price index using inflation rate or the price of oil as external transition variables.

As for the two transition variables, we can notice a pattern of complementarity in their dynamics in the sense that when both variables were employed in the STR model to characterize the nonlinearity in both the mean and the variance of the Grilli & Yang commodity price index, inflation was capable of capturing the early dynamics (between 1900 and 1950) of the index whereas oil price captured the late ones (between 1980 and 2007). The reason why inflation failed to account for the observed recent fluctuations in the GYCPI is, perhaps, due to the continuous monitoring of the FED to the U.S. inflation rate as part of its monetary policy. This result motivates the use of *external* threshold variables in the smooth transition regression model in general and, in particular, the use of inflation and oil price in the smooth transition model when applied to commodity prices. It should be noted, however, that the use of external threshold variable is advisable as long as it can be justified, i.e., as long as there exists a relationship between the threshold variable and the transition variable under consideration.

References

- [1] Akaike H. 1974. A new look at the statistical model identification. *IEEE Transactions on Automatic Control* **AC-19**: 716-723.

- [2] Ardeni PG, Wright B. 1992. The Prebisch-Singer hypothesis: a reappraisal independent of stationarity hypothesis. *Economic Journal* **102**: 803-812.
- [3] Bleaney M, Greenaway D. 1993. Long-run trends in the relative price of primary commodities and in the terms of trade of developing countries. *Oxford Economic Papers* **45**: 349-363.
- [4] Blomberg BS, Haris ES. 1995. The commodity-consumer prices connection: fact or fable? *Federal Reserve Bank of New York Economic Policy Review* **1**(3): 21-38.
- [5] Boughton JM, Branson WH. 1991. Commodity prices as a leading indicator of inflation. *NBER Working Paper Series No. 2750*.
- [6] Cuddington JT, Urzua CM. 1989. Trends and cycles in the net barter terms of trade: a new approach. *Economic Journal* **99**: 426-442.
- [7] Danthine JP. 1977. Martingale, market efficiency and commodity prices. *European Economic Review* **10**: 1-17.
- [8] Davies RB. 1977. Hypothesis testing when a nuisance parameter is present only under the alternative. *Biometrika* **64**: 247-254.
- [9] Deaton A, Laroque G. 1992. On the behavior of commodity prices. *Review of Economic Studies* **59**: 1-24.
- [10] Dickey DA, Fuller WA. 1979. Distributions of the estimators for autoregressive time series with a unit root. *Journal of The American Statistical Association* **74**: 427-431.
- [11] Eitrheim Ø, Teräsvirta T. 1996. Testing the adequacy of smooth transition autoregressive models. *Journal of Econometrics* **74**: 59-75.

- [12] Engle RF. 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica* **50**: 987-1007.
- [13] Fahmy H. 2011. *Regime switching in commodity prices* (thesis). Concordia University, Montreal, Canada.
- [14] Frankel JA. 1986. Expectations and commodity price dynamics: the overshooting model. *American Journal of Agricultural Economics* **68**: 344-348.
- [15] Fuhrer J, Moore G. 1992. Monetary policy rules and the indicator properties of asset prices. *Journal of Monetary Economics* **29(2)**: 303-336.
- [16] Granger C, Teräsvirta T. 1993. *Modelling nonlinear economic relationships*. Oxford University Press: Oxford.
- [17] Grilli ER, Yang C. 1988. Primary commodity prices, manufactured goods prices, and the terms of trade of developing countries: What the long run shows. *World Bank Economic Review* **2**: 1-47.
- [18] Gustafson RL. 1958. *Carryover levels for grains*. U.S.D.A. Technical Bulletin No. 1178.
- [19] Helg R. 1991. A note on the stationarity of the primary commodities relative price index. *Economics Letters* **36**: 55-60.
- [20] Jarque CM, Bera AK. 1987. A test for normality of observations and regression residuals. *International Statistical Review* **55**: 163-172.
- [21] Kwiatkowski D, Phillips PCB, Schmidt P, Shin Y. 1992. Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of Econometrics*, **54**: 159-178.

- [22] Kyrtsoy C, Labys WC. 2006. Evidence for chaotic dependence between US inflation and commodity prices. *Journal of Macroeconomics*, **28**: 256-266.
- [23] Lee J, Degennaro RP. 2000. Smooth Transition ARCH Models: Estimation and Testing. *Review of Quantitative Finance and Accounting*, **15**: 5-20.
- [24] Ljung GM, Box GEP. 1978. On a measure of lack of fit in time-series models. *Biometrika* **65**: 297-303.
- [25] Lubrano M. 2001. Smooth Transition GARCH Models: A Bayesian Perspective. *Recherches Economiques de Louvain* **67**: 257-287.
- [26] Luukkonen R, Saikkonen P, Teräsvirta T. 1988. Testing linearity against smooth transition autoregressive models. *Biometrika* **75**: 491-499.
- [27] Lundbergh S, Teräsvirta T. 1998. Modelling Economic High Frequency Time Series with STARSTGARCH Models. *SSE/EFI Working Paper Series in Economics and Finance* **No. 291**.
- [28] Lutz MG. 1999. A general test of the Prebisch-Singer hypothesis. *Review of Development Economics* **3**: 44-57.
- [29] Muth JF. 1961. Rational expectations and the theory of price movements. *Econometrica* **29**: 315-35.
- [30] Newbold P, Vougas D. 1996. Drift in the relative prices of primary commodity prices: A case where we care about unit roots. *Applied Economics* **28**: 653-661.
- [31] Persson A, Teräsvirta T. 2003. The net barter terms of trade: a smooth transition approach. *International Journal of Finance and Economics* **8**: 81-97.

- [32] Pfaffenzeller S, Newbold P, Rayner A. 2007. A short note on updating the Grilli-Yang commodity price index. *World Bank Economic Review* **21(1)**: 151-163.
- [33] Phillips PCB, Perron P. 1988. Testing for a unit root in time series regression. *Biometrika*, **75**: 335-346.
- [34] Prebisch R. 1950. The economic development of Latin America and its principal problems. *Economic Bulletin For Latin America* **7**: 1-12.
- [35] Samuelson PA. 1971. Stochastic speculative price. *Proceedings of the National Academy of Sciences* **68**: 335-337.
- [36] Singer H. 1950. The distributions of gains between investing and borrowing countries. *American Economic Review* **40**: 473-485.
- [37] Teräsvirta T. 1994. Specification, estimation, and evaluation of smooth transition autoregressive models. *Journal of American Statistical Association* **89**: 208-218.
- [38] Teräsvirta T. 1998. Modeling economic relationships with smooth transition regressions, in Ullah A, Giles DE (eds.). *Handbook of Applied Economic Statistics*, Dekker, New York, pp. 507-552.
- [39] van Dijk D, Teräsvirta T, Franses PH. 2002. Smooth transition autoregressive models - A survey of recent developments. *Econometric Reviews* **21(1)**:1-47
- [40] Von Hagen J. 1989. Relative commodity prices and cointegration. *Journal of Business & Economic Statistics* **7(4)**: 497-503.
- [41] Williams JC, Wright BD. 1991. *Storage and commodity markets*. Cambridge University Press, Cambridge.

Table 1: The stationarity of the commodity price index, inflation, and the price of oil.

Time Series	ADF(p)	PP	KPSS _τ
$y_t = \log\left(\frac{GYCPI_t}{MUUV_t}\right)$	-3.99(1)	-3.75	0.13
$\Pi_t = \Delta \log(CPI_t)$	-5.65(1)	-5.25	0.07
$R_t = \log\left(\frac{Oil\ price_t}{CPI_t}\right)$	-3.11(0)	-3.15	0.14

ADF, PP, and KPSS are respectively the augmented Dickey-Fuller (1979), the Phillips-Perron (1988), and the Kwiatkowski, Phillips, Schmidt, and Shin (1992) tests with a trend. P is the number of lags of the ADF test. The 5% critical values are -3.45 for ADF and PP and 0.146 for KPSS.

Table 2: P-values of the linearity F-tests sequence applied to the logarithm of the real GY commodity price index when inflation is the transition variable.

s_t	F_L	F_4	F_3	F_2	Suggested Model
y_{t-1}	7.31×10^{-1}	4.50×10^{-1}	6.48×10^{-1}	4.74×10^{-1}	Linear
Δy_{t-1}	2.39×10^{-1}	8.29×10^{-1}	6.45×10^{-2}	3.40×10^{-1}	Linear
Π_t^{**}	1.06×10^{-4}	2.08×10^{-2}	3.02×10^{-4}	1.57×10^{-1}	LSTR(2)
Π_{t-1}^*	5.08×10^{-3}	6.17×10^{-1}	8.56×10^{-1}	1.53×10^{-4}	LSTR(1)

Table 3: P-values of the linearity F-tests sequence applied to the logarithm of the real GY commodity price index when the logarithm of real oil is the transition variable.

s_t	F_L	F_4	F_3	F_2	Suggested Model
y_{t-1}	7.31×10^{-1}	4.50×10^{-1}	6.48×10^{-1}	4.74×10^{-1}	Linear
Δy_{t-1}	2.39×10^{-1}	8.29×10^{-1}	6.45×10^{-2}	3.40×10^{-1}	Linear
R_t	1.12×10^{-1}	6.18×10^{-2}	5.00×10^{-1}	1.90×10^{-1}	Linear
R_{t-1}^*	4.66×10^{-4}	1.27×10^{-2}	8.29×10^{-2}	4.98×10^{-3}	LSTR(1)

Table 4: Comparison between the STR in mean and the STR in variance model with one period lag inflation rate as the transition variable.

	STR in mean model	STR in variance model
Threshold value	$\Pi_{t-1} = 0.07$	$\Pi_{t-1} = 0.03$
Upper Regime:		
Equation	$y_t = 0.009 + 0.54y_{t-1} + \hat{\varepsilon}_t$	$y_t = 0.001 + 0.97y_{t-1} + \hat{\xi}_t$
Error variance	$\sigma^2 = 0.01$	$h_t = 0.02$
Behavior of y_t	Stationary AR(1)	Near Random Walk
Lower Regime:		
Equation	$y_t = 0.009 + 0.98y_{t-1} + \hat{\varepsilon}_t$	$y_t = 0.001 + 0.97y_{t-1} + \hat{\xi}_t$
Error Variance	$\sigma^2 = 0.01$	$h_t = 0.006$
Behavior of y_t	Near Random Walk	Near Random Walk

Table 5: Comparison between the STR in mean and the STR in variance model with the one period lag logarithm of real oil price as the transition variable.

	STR in mean model	STR in variance model
Threshold: s_t	$R_{t-1} = -3.1$	$R_{t-1} = -3.1$
Upper Regime:		
Equation	$y_t = -0.09 + 0.58y_{t-1} + \hat{\varepsilon}_t$	$y_t = 0.005 + 0.95y_{t-1} + \hat{\xi}_t$
Error Variance	$\sigma^2 = 0.01$	$h_t = 0.03$
Behavior of y_t	Stationary AR(1)	Near Random Walk
Lower Regime:		
Equation	$y_t = 0.009 + 0.98y_{t-1} + \hat{\varepsilon}_t$	$y_t = 0.005 + 0.95y_{t-1} + \hat{\xi}_t$
Error Variance	$\sigma^2 = 0.01$	$h_t = 0.008$
Behavior of y_t	Near Random Walk	Near Random Walk

Table 6: The border price, the model type, and the transition variable for each individual commodity in the Grilli and Yang data set.

Series	Border Price	Model	Transition Variable: s_t
Maize	FOB	STR	Inflation
Rice	FOB	STR	Inflation
Sugar	FOB	STR	Inflation
Bananas	CIF	STR	Oil Price
Timber	CIF	STR	Oil Price
Palm oil	CIF	STR	Oil Price
Coffee	CIF	STR	Oil Price

Modelling nonlinearities in commodity prices using smooth transition regression models with external transition variables

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Abstract

This paper investigates the nonlinearities in commodity prices using smooth transition regression (STR) models. The STR model, which is technically a nonlinear model in the conditional mean, is extended to model nonlinearities in the conditional variance equation of the price series. What distinguishes this paper from the majority of the studies in the smooth transition literature is its use of external transition variables, in addition to the standard autoregressive lags of the dependent variable, in modelling the regime switching behavior of commodity prices. Two external transition variables were found successful in capturing the regime switching dynamics of commodity prices: inflation rate and the price of oil. When both variables were used in the STR in mean and the STR in variance models, they displayed the same dynamics in their limiting

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